Seasonality Tests

Fabio Busetti and Andrew Harvey Bank of Italy and
Faculty of Economics and Politics, University of Cambridge

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Abstract

This article modifies and extends the test against nonstationary stochastic seasonality proposed by Canova and Hansen (1995). A simplified form of the test statistic in which the nonparametric correction for serial correlation is based on estimates of the spectrum at the seasonal frequencies is considered and shown to have the same asymptotic distribution as the original formulation. Under the null hypothesis the distribution of the seasonality test statistics is not affected by the inclusion of trends, even when modified to allow for structural breaks, or by the inclusion of regressors with nonseasonal unit roots. A parametric version of the test is proposed and its performance is compared with that of the nonparametric test using Monte Carlo experiments. A test which allows for breaks in the seasonal pattern is then derived. It is shown that its asymptotic distribution is independent of the break point and its use is illustrated with a series on UK marriages. A general test against any form of permanent seasonality, deterministic or stochastic, is suggested and compared with a Wald test for the significance of fixed seasonal dummies. It is noted that tests constructed in a similar way can be used to detect trading day effects. An appealing feature of the proposed test statistics is that under the null hypothesis they all have asymptotic distributions belonging to the Cramér-von Mises family.

KEYWORDS: Cramér-von Mises distribution, Locally best invariant test, Seasonal breaks, Structural time series model, Trading day effects, Unobserved components.

JEL classification: C22, C52

Monthly and quarterly economic time series are often subject to seasonal movements. These seasonal patterns tend to evolve over time and most seasonal adjustment procedures assume that this is the case. However, if the seasonal pattern does not change, it can be modelled by a set of dummy variables. Indeed a deterministic seasonal pattern can be removed without even estimating a time series model. All that needs to be assumed is the number of times the series needs to be differenced to make it stationary; see Pierce (1978) and Harvey (1989, p 203). Adjusting series in this way may simplify the exploration of relationships between time series.

Canova and Hansen (1995)- hereafter CH- proposed a test of the null hypothesis that the seasonal pattern is deterministic against the alternative that it evolves as a nonstationary stochastic process. The test includes a nonparametric correction for serial correlation and seasonal heteroscedasticity. The aim of this paper is to extend the CH test in various directions. We show how to modify the test so as to allow for the effect of modeling breaks in the seasonal pattern. We then consider a different, but related testing problem, namely testing for the presence of any kind of seasonal effects, whether they be deterministic, stochastic or both. Similar techniques can be used for detecting trading day effects. Before describing these extensions, we examine the CH test in more detail, look at some alternative formulations and compare the performance of the nonparametric test with a parametric one.

Section 1 shows how a nonparametric correction for serial correlation can be set up in terms of the spectrum at seasonal frequencies. This formulation is more restrictive than the CH test insofar as it does not allow for seasonal heteroscedasticity. Subject to this proviso, its asymptotic distribution under the null hypothesis is the same as in the original formulation, but its interpretation is more transparent. In deriving asymptotic distributions we relax the conditions stated in Canova and Hansen (1995) by showing that the distribution is unaffected when a deterministic trend is included in the model; regressors with unit roots can also be included provided they do not have seasonal unit roots.

Parametric versions of the tests against nonstationary seasonality can be based on structural time series models. Such models are set up in terms of unobserved components, such as trends and cycles, which have a direct interpretation; see Harvey (1989) and Kitagawa and Gersch (1996). The use of autoregressive models, as in Caner (1998), is less appealing in this context, since they may yield a poor approximation to the moving average terms typically found in the reduced forms of structural models; see also Taylor (2002a). The evidence in Leybourne and McCabe (1994) and Harvey and Streibel (1997) suggests that when testing against the presence of a random walk component, a parametric approach will usually give tests with a higher power and more reliable size. We investigate whether this is the case for seasonality tests by a series of Monte Carlo experiments. The results are reported in the final sub-section. As well as casting light on the relationship between parametric and nonparametric tests, they provide information on the robustness of the nonparametric test to the order of differencing.

Breaks in the trend leave the asymptotic distribution unaffected if they are correctly modelled by the inclusion of dummy variables; this is proved in section 1. Structural breaks in the seasonal pattern are also considered. Empirical results in Ghysels (1990) and Canova and Ghysels (1994) suggest that seasonal mean shifts are not uncommon in US quarterly series. Neglecting these shifts will bias the nonstationarity tests at both the zero and the seasonal frequencies. However, modeling breaks in the seasonal pattern will affect the distribution of the seasonality test statistics. Section 2 shows how to construct a test statistic against stochastic seasonality, the asymptotic distribution of which is independent of the breakpoint location.

The parametric and nonparametric tests, with the breakpoint modification, are illustrated by an application to a quarterly series on UK marriages. The point about this example is that there is an identifiable break in the seasonal pattern because of a known change in the tax laws. The modified test is trying to assess if it is necessary to allow for stochastic seasonality once the deterministic break has been accounted for by intervention dummy variables.

Section 3 suggests a general test for seasonality. This takes the same form as the test against nonstationary seasonality, except that seasonal dummies are not fitted. The asymptotic distribution is given and the performance of the test is compared with that of a Wald test for the significance of fixed seasonal dummies. Similar techniques are used to construct a test for the presence of trading day effects.

A unifying feature of the test statistics is that, under the null hypothesis, they all have asymptotic distributions that belong to the Cramér-von Mises family. These distributions differ according to the deterministic components fitted and a degrees of freedom parameter. The same distributions arise in tests against nonstationary trends as noted in Harvey (2001).

1. Testing against the Presence of a Nonstationary Seasonal Component

This section develops the trigonometric form of the test against nonstationary stochastic seasonality, shows that it is locally best invariant, gives nonparametric corrections for serial correlation and shows how to set up a parametric test. Monte Carlo simulation experiments are then used to compare the performance of the parametric and nonparametric tests in small samples.

1.1. The Testing Framework

Let y_t be a scalar time series, denote by s the number of seasons in a year and let $Z_t = \left(z'_{1t}, ..., z'_{[s/2]t}\right)'$ be an $(s-1)$-vector of trigonometric seasonal variables, that is $z_{jt} = (\cos 2\pi jt/s, \sin 2\pi jt/s)'$, $j = 1, ..., s^*$, where $s^* = s/2 - 1$ if s is even or $[s/2]$ if s is odd, while $z_{s/2,t} = (-1)^t$ if s is even. The j-th pair of trigonometric terms, z_{jt}, corresponds to the j-th harmonic seasonal frequency, $\lambda_j \equiv 2\pi j/s$, $j = 1, ..., s^*$. When s is even, the last element of Z_t, $z_{[s/2]t}$, corresponds to the Nyquist frequency, $\lambda_{[s/2]} \equiv \pi$.

The test against stochastic seasonality is developed in the context of the following unobserved components model

$$\begin{aligned} y_t &= \mu_t + s_t + \varepsilon_t, \qquad t = 1, ..., T, & (1.1) \\ \mu_t &= X'_t\beta, & (1.2) \\ s_t &= Z'_t\gamma_t, & (1.3) \\ A'\gamma_t &= A'\gamma_{t-1} + \kappa_t, & (1.4) \end{aligned}$$

where X_t is a $p \times 1$ vector of linearly independent deterministic regressors with associated coefficient vector β, s_t is a time varying seasonal component with Z_t defined as above, A is a known $(s-1) \times k$ selection matrix with rank $k \leq s-1$, and ε_t and κ_t are mutually uncorrelated mean zero disturbances with variances σ^2_ε and $\sigma^2_\kappa W$ respectively. The initial value γ_0 is assumed to be fixed.

The aim is to test the null hypothesis that the seasonal component is deterministic, $\mathrm{H}_0 : \sigma^2_\kappa = 0$, against the alternative that it has unit root behavior, $\mathrm{H}_1 : \sigma^2_\kappa > 0$. Following Canova and Hansen (1995), the matrix A is used to formulate tests at subsets of the seasonal frequencies $\lambda_1, ..., \lambda_{[s/2]}$. If the test is to be carried out at the single frequency λ_j, we will let A_j denote the $(2j-1)^{th}$ and $(2j)^{th}$columns of I_{s-1} for $j < s/2$ and the $(2j-1)^{th}$ column of I_{s-1} if $j = s/2$; I_k denotes an identity matrix of dimension k. When all the seasonal frequencies are considered $A = I_{s-1}$.

1.2. Locally best invariant test

Under Gaussianity, the locally best invariant (LBI) test for the null hypothesis of deterministic seasonality for the model (1.1)-(1.4) can be easily obtained from the results of King and Hillier (1985), where it is also shown that the test is a one-sided LM test, and Taylor (2002a).

Specifically, let e_t, $t = 1, ..., T$, be the ordinary least squares (OLS) residuals from regressing y_t on $(X_t', Z_t')'$ and let $\widehat{\sigma}^2 = T^{-1}\sum_{t=1}^{T} e_t^2$ be their sample variance. Let $a_j = 1$ if $j = s/2$ and $a_j = 2$ otherwise.

Consider first each seasonal frequency λ_j, $j = 1, ..., [s/2]$, in turn, that is $A = A_j$ in the model (1.1)-(1.4). Then, under Gaussianity, the LBI test for testing $\text{H}_0 : \sigma_\kappa^2 = 0$ against $\text{H}_1 : \sigma_\kappa^2 > 0$ rejects for large values of the statistic ω_j, defined as

$$\omega_j = a_j T^{-2}\widehat{\sigma}^{-2}\sum_{t=1}^{T}\left[\left(\sum_{i=1}^{t} e_i \cos\lambda_j i\right)^2 + \left(\sum_{i=1}^{t} e_i \sin\lambda_j i\right)^2\right], \qquad j = 1, ..., [s/2]. \tag{1.5}$$

Note that, when s is even, the test statistic at the Nyquist frequency, $\omega_{s/2}$, can be written without the terms $e_i \sin\lambda_{s/2} i$ as they are identically zero.

A complete test against nonstationary seasonality at all frequencies, that is $A = I_{s-1}$, rejects for large values of the statistic obtained by adding up the test statistics for each individual frequency, namely

$$\omega = \sum_{j=1}^{[s/2]} \omega_j, \tag{1.6}$$

This test is LBI for a model where, under the alternative hypothesis, the coefficients corresponding to each seasonal frequency λ_j, $j = 1, ..., s^*$, evolve as mutually independent random walks with variances σ_κ^2 and, if s is even, the coefficient for frequency π is a random walk with variance $\sigma_\kappa^2/2$. Thus W is an identity matrix unless s is even in which case the last element in the main diagonal is 1/2.

Under H_0, $\omega_j \xrightarrow{d} CvM(a_j)$ and $\omega \xrightarrow{d} CvM(s-1)$, where $CvM(k)$ denotes a Cramér-von Mises random variable with k degrees of freedom, and $\sum_{j=1}^{[s/2]} a_j = s - 1$. Gaussianity of the $\varepsilon_t's$ is not needed; all that is required is that they be martingale differences satisfying the conditions in Andrews (1991, p 823) or Stock (1994, p.2745). The proof is a special case of proposition 1.1 below.

1.3. Nonparametric Correction for Serial Correlation Based on the Spectrum at Seasonal Frequencies

Serial correlation in the stationary component of (1.1)-(1.4) can be treated nonparametrically by replacing the sample variance, $\widehat{\sigma}^2$, in ω_j with an estimator of the spectrum of ε_t at frequency λ_j. We denote this *spectral nonparametric* test statistic as

$$\omega_j(m) = \frac{a_j \sum_{t=1}^{T} \left[\left(\sum_{i=1}^{t} e_i \cos \lambda_j i \right)^2 + \left(\sum_{i=1}^{t} e_i \sin \lambda_j i \right)^2 \right]}{T^2 \widehat{g}(\lambda_j; m)}, \qquad j = 1, ..., [s/2], \tag{1.7}$$

where

$$\widehat{g}(\lambda_j; m) = \sum_{\tau=-m}^{m} w(\tau, m) \widehat{\gamma}_e(\tau) \cos \lambda_k \tau \tag{1.8}$$

is the estimator of the spectral generating function, $w(\tau, m)$ is a weighting function or kernel, such as $w(\tau, m) = 1 - |\tau|/(m+1)$, and $\widehat{\gamma}_e(\tau) = T^{-1} \sum_{t=\tau+1}^{T} e_t e_{t-\tau}$ is the sample autocovariance of the OLS residuals at lag τ. Alternative options for the kernel $w(.,.)$ may be found in Andrews (1991, p.821). For testing all the seasonal frequencies the spectral nonparametric statistic is

$$\omega(m) = \sum_{j=1}^{[s/2]} \omega_j(m).$$

Under the assumptions set out below, the asymptotic distributions of the above test statistics under the null hypothesis will be the same as in the previous sub-section.

Assumption A1. X_t is a $p \times 1$ vector of deterministic regressors and D_T is a (diagonal) scaling matrix such that

$$\begin{array}{ll} (i) & \lim_{T \to \infty} T^{-1} \sum_{t=1}^{T} D_T^{-1} X_t X_t' D_T^{-1} = Q_x, \\ (ii) & \lim_{T \to \infty} T^{-1} \sum_{t=1}^{T} D_T^{-1} X_t Z_t' = 0, \end{array}$$

where Q_x is a positive definite matrix.

Assumption A2. The $\varepsilon_t' s$ have the structure of a linear process, $\varepsilon_t = \psi(L) \varepsilon_t^*$, where $\{\varepsilon_t^*\}$ is a martingale difference sequence satisfying the conditions in Stock

(1994, p.2745), and $\psi(L) \equiv 1 + \sum_{i=1}^{\infty} \psi_i L^i$ is a polynomial in L, the conventional lag operator, $L^k y_t \equiv y_{t-k}$, $k = 0, 1, ...$, satisfying: (i) $\psi(\exp\{\pm i2\pi\lambda_j/s\}) \neq 0$, for all $j = 1, ..., [s/2]$, and (ii) $\sum_{k=1}^{\infty} k|\psi_k| < \infty$.

Assumption A3. The lag truncation parameter, m, is such that, as $T \to \infty$, $m \to \infty$ and $m/T^{1/2} \to 0$; see Stock (1994, p. 2797-2799).

Proposition 1.1. *Let y_t be generated by the model (1.1)-(1.4) under the following assumptions A1-A3. Then, under $H_0 : \sigma_\kappa^2 = 0$, when $A = A_j$, $j = 1, ..., [s/2]$,*

$$\omega_j(m) \xrightarrow{d} \int_0^1 B_{a_j}(r)' B_{a_j}(r) \equiv CvM(a_j),$$

when $A = I_{s-1}$, $\omega(m) \xrightarrow{d} CvM(s-1)$, where $B_k(r) = W_k(r) - rW_k(1)$, and $W_k(r)$, $r \in [0, 1]$, denotes a k-dimensional standard Wiener process. Under $H_A : \sigma_\kappa^2 > 0$ and when $A = A_j$, $j = 1, ..., [s/2]$, $\omega_j(m)$ and $\omega(m)$ are $O_p(T/m)$.

Proposition 1.1 extends the result given in Canova and Hansen (1995) as it allows for a general specification for the deterministic trend μ_t; in their article μ_t was a constant level. The limiting distribution is unchanged provided the regressors X_t satisfy assumption A1. In particular, contrary to what Canova and Hansen (1995, p238) state, the model can include linear trends. Structural breaks in the trend at known points can also be included. Thus if $X_t = (1, t, d_t(\alpha))$, where $d_t(\alpha)$ is a dummy variable equal to one for $t > \alpha T$,with $0 < \alpha < 1$, and equal to zero for $t \leq \alpha T$ assumption A1 is satisfied by choosing $D_T = diag(1, T, 1)$.

As concerns the properties of the disturbances, ε_t, the first condition of assumption A2 rules out a zero in the spectrum at any of the seasonal frequencies λ_j, $j = 1, ..., [s/2]$, while the second ensures that poles do not exist in the spectrum. These conditions are satisfied by any finite-order stationary and invertible ARMA processes. Assumption A3 is required to achieve consistency of the test under the (fixed) alternative $\text{H}_A : \sigma_\kappa^2 > 0$.

Model (1.1)-(1.4) can be extended by including stochastic regressors with non-seasonal unit roots and it can be shown, following the line of the proof of proposition 1.1, that the asymptotic critical values for the tests in the augmented model are unchanged; the proof is omitted but available from the authors on request. Such regressors are ruled out by Canova and Hansen (1995) who state that '....the explanatory variables may be any non-trending variables that satisfy weak dependence conditions'. The generalisation is of some practical importance. Note that the presence of cross correlation between ε_t and the disturbance vector driving the stochastic regressors is not important for our testing seasonality; unless we are interested in making inferences on the coefficient vector of the regressors there is no need to use, say, a fully modified least squares procedure instead of OLS.

In summary, the inclusion of deterministic trends and stochastic integrated regressors does not affect the asymptotic distribution of the seasonal test statistics. However, the inclusion of seasonal slopes will affect the distribution, just as it does for tests of seasonal unit roots as discussed by Smith and Taylor (1998) and Taylor (2002a). Given the obvious parallels with stationarity tests, it is not difficult to see that the change in distribution will be the same as when a time trend is fitted before a stationarity test statistic is constructed. The critical values, which are once again from the Cramér-von Mises family, are as in table 2 of Nyblom and Harvey (2000). For quarterly data the 5% critical value for an overall test, based on three degrees of freedom, is[1] 0.337.

Dummy variables introduced to capture breaks in the seasonal pattern will also affect the distribution of test statistics; this is the subject of section 2.

1.4. The Canova-Hansen Test Statistic

The test statistic proposed by Canova and Hansen (1995) in the framework of (1.1)-(1.4) takes the form

$$\omega_A(m) = T^{-2} trace\left(\left(A'\widehat{\Omega}(m)A\right)^{-1} A' \sum_{t=1}^{T} S_t S_t' A\right), \tag{1.9}$$

where $S_t = \sum_{i=1}^{t} Z_i e_i$ and $\widehat{\Omega}(m)$ is a nonparametric estimator of the "long run variance" of $Z_t\varepsilon_t$, that is

$$\widehat{\Omega}(m) = \sum_{\tau=-m}^{m} w(\tau, m)\widehat{\Gamma}(\tau), \tag{1.10}$$

where $w(\tau, m)$ is a kernel as in (1.8), and $\widehat{\Gamma}(\tau) = T^{-1}\sum_{t=\tau+1}^{T} Z_t e_t e_{t-\tau} Z_{t-\tau}'$ is the sample autocovariance matrix at lag τ formed from $Z_t e_t$. The main difference between (1.9) and $\omega(m)$ is that (1.9) allows for seasonal heteroscedasticity as in Andrews (1991, p839); see Canova and Hansen (1995, page 240). Under the null

[1] There is actually a typographical error in the published table, with the figure being given as 0.332.

hypothesis $H_0 : \sigma^2_\kappa = 0$, the asymptotic representation of (1.9) corresponds to that of proposition 1.2, that is $\omega_A(m) \xrightarrow{d} CvM(rank(A))$.

1.5. Parametric tests

A structural time series model typically contains stochastic trend and seasonal components[2], together with an irregular. It can be extended in various ways, for example by including a cycle. However, for many economic time series, the flexibility of the stochastic trend is such that the model can still adequately capture seasonal movements if a cycle is excluded. We now consider how to set up a parametric test of whether the seasonal component in a structural time series model is stochastic.

If the process generating the non-seasonal part of the model is known, the LBI test against stochastic seasonality is constructed from a set of 'smoothing errors'. As shown in Appendix B the smoothing errors are, in general, serially correlated but the form of this serial correlation may be deduced from the specification of the model, thereby allowing the construction of a statistic that has a Cramér-von Mises distribution, asymptotically, under the null hypothesis. The computation of smoothing errors is discussed in de Jong and Penzer (1998), but if the model contains a serially uncorrelated irregular component, it can be shown that they are proportional to the optimal estimates of this component.

An alternative possibility is to use the T standardized one-step ahead prediction errors, the innovations, calculated by treating nonstationary and deterministic components as having fixed initial conditions. No correction is then needed; the statistic is of the form (1.5) and has the same asymptotic distribution. Note that calculating innovations under the assumption that the initial conditions are fixed requires that the initial conditions be estimated, but that a backward smoothing recursions can be avoided simply by reversing the order of the observations and calculating a set of innovations starting from the filtered estimator of the state at the end of the sample[3].

[2] The model underlying the LBI trigonometric seasonality test based on ω is one in which the seasonal regressors have coefficients which follow random walks under the alternative and the disturbances driving the random walks are mutually uncorrelated white noise processes with variances σ^2_ω at seasonal frequencies less than π, and, for s even, a variance of $\sigma^2_\omega/2$ at π. This model can be transformed to the one in Harvey (1989, ch 2) except that there the variance of the disturbance driving the component at frequency π is not divided by two. Proietti (2000) gives convincing reasons, unconnected with the LBI test, as to why division by two is preferable.

[3] The forward and backward innovations are not the same. Furthermore in neither case

For both the smoothing error and innovation forms of the test, nuisance parameters will normally have to be estimated. For stationarity tests, Leybourne and McCabe (1994) argue that this is best done under the alternative using maximum likelihood. Proceeding in this way, has the compensating advantage that since there will often be some doubt about a suitable model specification, estimation of the unrestricted model affords the opportunity to check its suitability by the usual diagnostics and goodness of fit tests. Once the nuisance parameters have been estimated, the test statistic is calculated from the innovations obtained with σ_κ^2 set to zero. The asymptotic distribution under the null is unaffected.

1.6. Monte Carlo Experiments

This sub-section compares the probability of rejecting the null hypothesis of constant seasonality using the spectral nonparametric test and a parametric test based on a correctly specified model. The results offer guidance in assessing the effectiveness of the two approaches as well as establishing the reliability of the tests in terms of actual, as opposed to nominal, size.

The data generation process is the basic structural model (BSM), consisting of seasonal and stationary components combined with a local linear trend, μ_t. Thus,

$$y_t = \mu_t + \gamma_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2), \qquad t = 1, ..., T \tag{1.11}$$

where

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t, \quad \eta_t \sim NID(0, \sigma_\eta^2) \\ \beta_t &= \beta_{t-1} + \zeta_t, \quad \zeta_t \sim NID(0, \sigma_\zeta^2) \end{aligned}$$

with γ_t as in (1.4) with $A = I$.

The probability of rejection depends on the seasonal signal-noise ratio, $q_\kappa = \sigma_\kappa^2/\sigma_\varepsilon^2$, though in the tables we prefer to report the square root. The results are for quarterly series of length $T = 200$ and the rejection frequencies, reported in percentages, are based on 50,000 replications. The program was written in Ox using the SSFpack set of subroutines of Koopman, Shephard and Doornik (1999).

The first set of experiments is for a model with $\sigma_\zeta^2 = 0$ so the trend is a random walk with constant drift. The performance of the tests does not depend

do the sums, weighted by $\cos \lambda_j t$ and $\sin \lambda_j t$, equal zero, so statistics formed from forward and backward sums are different. Smoothing errors do not suffer from these ambiguities. Fortunately the asymptotic properties are unaffected.

on the value of the drift, β, and so this can be equal to zero. The critical factor in the nuisance parameters is the level signal-noise ratio, $q_\eta = \sigma_\eta^2/\sigma_\varepsilon^2$ and $q_\eta^{1/2}$ is set to 0.1, 0.5 and 1.0 in tables 1,2 and 3 respectively. The spectral nonparametric test and the CH test of sections 1.3-1.4 are applied both to levels and to first differences with the lag truncation parameter[4], m, set to 4 and 8. The parametric test statistics are computed firstly assuming that q_η is known and then with σ_η^2 and σ_ε^2 estimated by maximum likelihood under the alternative hypothesis. Finally the local linear trend is estimated without imposing the constraint that the slope variance is zero; this is denoted LLT, that is local linear trend. The parametric test results are shown for innovations, computed starting from the smoothed estimator at the end of the sample, and for smoothing errors.

The main findings are as follows.

a) Although the sizes of the parametric test with q_η known are very close to the nominal, the test with estimated q_η is somewhat oversized at frequency π. The joint test for overall seasonality is similarly oversized with the actual sizes being around 0.09. It is interesting to note that the autoregression based tests reported by Caner (1998, table 1) display even more oversizing.

b) When q_η is known, there is a slight power advantage for a parametric test constructed from smoothing errors rather than innovations. However, when the q_η is estimated this advantage disappears as the size of the innovations test is a little closer to the nominal.

c) Estimating the extra parameter, $q_\zeta = \sigma_\zeta^2/\sigma_\varepsilon^2$ in the more general local linear trend model has no adverse effect on the performance of the parametric test; on the contrary, there is a slight improvement in the size in that it is closer to the nominal.

d) There is only a small fall in the rejection probabilities for the nonparametric tests in moving from $m = 4$ to $m = 8$. The seasonality test seems less sensitive to lag length than does the KPSS test; compare the comments in Canova and Hansen (1995, p246). However, m should not be set too small. Our simulations for the joint test with $m = 0$ (unreported in the tables) showed rejection frequencies of 14.5, 13.4 and 11.0 for the models in tables 1,2, and 3 respectively.

e) The nonparametric test in levels has a much lower probability of rejection than the corresponding test in first differences when $q_\eta^{1/2}$ is 0.5 or 1. This is due to the presence of a so-called *unattended unit root* at frequency zero when the tests are run in levels. In fact Busetti and Taylor (2002) and Taylor (2002b) demonstrate that, although consistent, the nonparametric tests would suffer from

[4] Canova and Hansen (1995) use 3 and 5 for $T = 50$ and 150 respectively.

a great loss of power if that unit root is not removed by taking first differences of the data. Indeed, they show that under the null hypothesis of deterministic seasonality the test statistics (1.5), (1.7), (1.9) all converge in probability to zero; this is also clear from tables 1-3.

f) The parametric tests show higher rejection frequencies, but any assessment of power has to be offset against the higher size.

g) The nonparametric tests perform relatively better than the results for stationarity tests (at the zero frequency) would suggest. This is because in the experiments reported in the literature, for example Leybourne and McCabe (1994), the process generating the stationary part of the model - typically a first-order autoregression - interferes with the unit root process. This problem does not arise with the data generating processes considered here and it would be unlikely to arise even if a first-order autoregressive process were to replace the white noise irregular.

The second group of experiments, reported in tables 4 and 5, is for the smooth trend model in which σ_η^2 is zero and the relevant signal-noise ratio is now q_ζ. The parametric model is estimated both with and without σ_η^2 set to zero. For economic time series, q_ζ is typically fairly small.

h) Although, in theory, the first difference operator should be applied twice to avoid the power loss induced by the presence of unattended unit roots, tables 4-5 indicate that for $q_\zeta^{1/2} = 0.1, 0.5$ there is significant loss if only first differences are taken. In fact, for $q_\zeta^{1/2} = 0.1$, first differences may be preferable and since $q_\zeta^{1/2}$ is often smaller than 0.1 in practice, basing tests on first differences may be a good strategy.

i) The conclusions with respect to size and power are similar to those reached for the random walk plus drift model. The parametric tests are somewhat oversized, but have a higher probability of rejection under the alternative. Again there is no disadvantage to fitting a more general local linear trend model in carrying out the parametric tests.

As the models used in the above simulations do not exhibit seasonal heteroscedasticity it is not surprising that the spectral nonparametric test performs slightly better than the CH test. However, once seasonal heteroscedasticity is present the situation changes. For example with a model consisting of a seasonal plus white noise with the variance in the four quarters being 1,3,5 and 7 respectively, the size of a $\omega(4)$ test for $T = 1000$ was estimated to be 0.080, while $\omega_A(4)$ was 0.038. Thus the CH size is closer to the nominal 0.05. However, with $q = 0.025$, the respective estimated probabilities of rejection were 0.458 and 0.662.

This is a rather extreme case, but it does illustrate the point that, when seasonal heteroscedasticity is present, the CH test is not only more robust with respect to size, but may also show a higher probability of rejection away from the null.

2. Deterministic Breaks in the Seasonal Pattern

In this section we consider testing against nonstationary stochastic seasonality when there is a break in the seasonal pattern at time $[\alpha T]$, $\alpha \in [0,1]$, that is we replace (1.3) with

$$s_t = Z_t'\gamma_t + d_t(\alpha)Z_t'\theta \tag{2.1}$$

where $d_t(\alpha) = 1(t > \alpha T)$ is a break dummy variable. The model now implies that the coefficients of the seasonal terms have changed from γ_t when $t \leq [\alpha T]$ into $\gamma_t + \theta$ when $t > [\alpha T]$. We focus on the nonparametric tests, though, of course, the same issues arise with the parametric versions.

Firstly, it is assumed that the breakpoint parameter α is known. The extension to a situation where the breakpoint is unknown is discussed in subsection 2.3.

When there is a break in the seasonal pattern the nonparametric test statistics $\omega_j(m)$, $\omega(m)$, $\omega_A(m)$, of the previous section have to be constructed using the OLS residuals from regressing y_t on $(X_t', Z_t', d_t(\alpha)Z_t')'$. Their asymptotic representations under the null hypothesis are no longer Cramér-von Mises with a_j, $s-1$, $rank(A)$ degrees of freedom respectively, but depend in a rather complicated way on the breakpoint parameter α. However a simple modification yields test statistics, the null limiting distributions of which are still Cramér-von Mises but with degrees of freedom equal to $2a_j, 2(s-1)$ and $2rank(A)$. This extends to the seasonal case the modification to the LBI test at frequency zero suggested by Busetti and Harvey (2001). The parametric test can also be modified along the same lines.

2.1. Modified Test with Seasonal Break

The modified seasonal break spectral nonparametric statistic for testing against nonstationary seasonality at frequency λ_j, is defined as

$$\omega_j^*(\alpha; m) = \frac{a_j \sum_{t=1}^T c_{jt} k_t}{\widehat{g}(\lambda_j; m)}, \qquad j = 1, ..., [s/2] \tag{2.2}$$

where

$$c_{jt} = \left(\sum_{i=1}^{t} e_i \cos \lambda_j i\right)^2 + \left(\sum_{i=1}^{t} e_i \sin \lambda_j i\right)^2,$$
$$k_t = [\alpha T]^{-2}(1 - d_t(\alpha)) + [(1-\alpha)T]^{-2} d_t(\alpha),$$

a_j and $\widehat{g}(\lambda_j; m)$ are defined as in the previous section, and e_t are the OLS residuals from regressing y_t on $(X_t', Z_t', d_t(\alpha)Z_t')'$. The corresponding statistic for the test at all frequencies is then $\omega^*(\alpha; m) = \sum_{j=1}^{[s/2]} \omega_j^*(\alpha; m)$. The modifications to the CH statistic, (1.9), and the parametric statistic are carried out in a similar way.

Proposition 2.1. *Let y_t be generated by the model (1.1),(1.2),(2.1),(1.4) under the assumptions A1-A3. Then, under $H_0 : \sigma_\kappa^2 = 0$, when $A = A_j$, $j = 1, ..., [s/2]$, $\omega_j^*(\alpha; m) \xrightarrow{d} CvM(2a_j)$,when $A = I_{s-1}$, $\omega^*(\alpha; m) \xrightarrow{d} CvM(2s-2)$. Under H_A : $\sigma_\kappa^2 > 0$ and when $A = A_j$, $j = 1, ..., [s/2]$, $\omega_j^*(\alpha; m)$ and $\omega^*(\alpha; m)$ are $O_p(T/m)$.*

The idea behind the construction of (2.2) is to combine the evidence in the two subsamples $\{1,...,[\alpha T]\}$ and $\{[\alpha T]+1,...,T\}$. Note that $\omega_j^*(0.5; m) = 0.25\omega_j(m)$; thus when the breakpoint is in the middle of the sample the tests defined by the two statistics are the same. This is important as the latter has properties of optimality, being obtained by extending the LBI/LM test to deal with serial correlation in the stationary component. Furthermore, for the case of testing at frequency zero, Busetti and Harvey (2001) have shown via simulation experiments that for $\alpha \neq 0.5$ the loss of power of the modified test with respect to the LBI test is not great.

2.2. UK Marriages

The quarterly series of marriages registered in the UK from 1958q1 to 1982q4 was extracted from various issues of the UK Monthly Digest of Statistics. It is shown in figure 1(a). The spectral nonparametric test statistic, $\omega(m)$, calculated from first differences, is 4.18, 2.74 and 2.11 for lags of 4, 8 and 12 respectively. This leads to a rejection of the null hypothesis as the 5% critical value for the $CvM(3)$ distribution is 1.00. The original CH statistic, $\omega_A(m)$, gave smaller values, namely 1.78, 1.20 and 0.96.

Estimating (1.11) with a random walk trend using the STAMP 6 program of Koopman *et al* (2000) gives

$$\widetilde{\sigma}_\varepsilon = 0.00 \qquad \widetilde{\sigma}_\eta = 1.61 \qquad \widetilde{\sigma}_\kappa = 2.69$$

with an equation standard error (the standard deviation of the innovations), $\widetilde{\sigma}$, of 7.91. The parametric test statistic, constructed from the Kalman filter innovations, is 6.96 which is a much firmer rejection of the null hypothesis than was given by the nonparametric test. The reason for the rejection can be seen from figure 1(a): there appears to be a break in the seasonal pattern at the beginning of 1969. This is very clear from the plot of the individual seasons in figure 1(b) where it can be seen that there was a switch from winter marriages to marriages in the spring quarter. This happened because of a change in the tax law. Up to the end of 1968 couples were allowed to claim the married persons tax allowance retrospectively for the entire year in which they married. As the tax year begins in April this arrangement provided an incentive to marry in the first quarter of the calendar year, rather than in the spring. The abolition of this rule led to a marked decrease in the number of weddings in quarter one and a compensating rise in quarter two.

Adding a set of three seasonal break dummy variables, starting in the first quarter of 1969, to take account of a complete change in the seasonal pattern leads to the following estimates of the parameters:

$$\widetilde{\sigma}_{\varepsilon} = 2.42 \qquad \widetilde{\sigma}_{\eta} = 1.59 \qquad \widetilde{\sigma}_{\kappa} = 1.36$$

with

$$Q(9,7) = 12.54 \qquad and \qquad \widetilde{\sigma} = 5.66$$

where $Q(P, f)$ is the Box-Ljung statistic based on P residual autocorrelations but with f degrees of freedom; see Koopman et al (2000). The $t-$statistics for the seasonal break dummies are -8.33, 7.58 and 2.09 respectively. There is a big reduction in the estimate of the seasonal parameter, σ_{κ}, which no longer needs to be such as to allow the stochastic seasonal model to accommodate the change, and the equation standard error, $\widetilde{\sigma}$, has fallen considerably.

The modified seasonal break nonparametric test statistics carried out on the residuals obtained from regressing first differences on seasonal means and the seasonal break dummies are 2.06, 1.69 and 1.58 for $m = 4, 8$ and 12 respectively for ω^* and 1.80, 1.57 and 1.50 for the CH form, ω_A^*. Thus for $m = 4$ and 8 the null of a constant seasonal pattern is rejected by the ω^* test at the 5% level of significance since the critical value for $CvM(6)$ is 1.69. However, the smaller values for ω_A^* only lead to a rejection for $m = 4$. The corresponding parametric test statistic, calculated from the Kalman filter innovations, is 2.42 giving a stronger indication that there is still stochastic seasonality present. This is backed up by the fact that estimating the model with a fixed seasonal gives a significant Box-Ljung statistic of $Q(9,8) = 22.38$ while the fourth order residual autocorrelation, $r(4)$, is 0.33.

2.3. Unknown breakpoint

If the breakpoint parameter α is unknown the two-step strategy of Busetti and Harvey (2002) for testing stationarity in the presence of a structural break can be adapted in a straightforward manner. The idea is to estimate the breakpoint under the null by minimizing, over α, the error variance of an OLS regression of y_t on $(X_t', Z_t', d_t(\alpha)Z_t')'$, i.e. $\widehat{\alpha} = \arg\min_{\alpha} T^{-1} \sum_{t=1}^{T} e_t^2$, where e_t are the OLS residuals. Bai (1997) shows that under the null hypothesis of deterministic seasonality this estimator is superconsistent, in the sense that it converges to the true value at rate T instead of the usual rate $T^{1/2}$. Therefore the null asymptotic distributions of $\omega_j^*(\widehat{\alpha}; m)$ and $\omega^*(\widehat{\alpha}; m)$ are the same as if the true value α were used, while under the alternative hypothesis the statistics diverge irrespective of the break date used due to the presence of the seasonal unit roots. Therefore running the seasonal break tests with an estimated breakpoint leads to an asymptotically valid procedure. Clearly, some loss of power with respect to the case of carrying out the tests with α known is to be expected. Busetti and Harvey (2002) use Monte Carlo simulation experiments to evaluate this power loss for the zero frequency stationarity tests.

3. Testing against a Permanent Seasonal Component

The test against nonstationary seasonal components takes the null hypothesis to be a model in which seasonality is deterministic. Sometimes we may wish to test whether there is any seasonality at all, irrespective whether it is deterministic or stochastic. One strategy, implemented in the STAMP package, is to fit a structural time series model and then to perform a test of significance on the seasonal coefficients as estimated at the end of the period. However, this has the disadvantage that it will indicate no seasonal effects in a situation where seasonality has become less pronounced over time. This is precisely the kind of behaviour noted by Canova and Hansen (1995, p 24-50) in their analysis of US macroeconomic series.

3.1. Tests and their power

Our aim is to develop tests that are powerful against the presence of deterministic seasonality and/or stochastic seasonality. The stochastic seasonality is taken to be nonstationary so that the effects are *permanent* in that forecasts of the seasonal pattern remain constant rather than dying away. We therefore consider the data generating process (1.1)-(1.4) with $A = I_{s-1}$ and split the seasonal component s_t into a deterministic and a stochastic part,

$$s_t = s_t^D + s_t^S, \tag{3.1}$$

$$s_t^D = Z_t'\gamma_0, \tag{3.2}$$

$$s_t^S = Z_t' \sum_{i=1}^{t} \kappa_i, \tag{3.3}$$

where γ_0 is a fixed coefficients and κ_t is a zero mean IID sequence with variance $\sigma_\kappa^2 W$, independent of ε_t. The null hypothesis of no seasonality is

$$H_0 : \gamma_0 = 0,\ \sigma_\kappa^2 = 0$$

while the alternative hypotheses are of deterministic seasonality

$$H_1^D : \gamma_0 \neq 0,\ \sigma_\kappa^2 = 0$$

and stochastic seasonality

$$H_1^S : \gamma_0 = 0,\ \sigma_\kappa^2 > 0.$$

We will show in the proposition below that the standard Wald test on fixed seasonal coefficients is consistent against both alternative hypotheses. We also show that a test constructed in a similar way to the tests of section 1, but without fitting seasonal regressors, is consistent against both hypotheses. More specifically let e_t be the OLS residuals from regressing y_t on $(X_t', Z_t')'$ and $\underline{e}_t$ the residual from regressing y_t on X_t only. Since Assumption A1 requires that the two sets of regressors X_t and Z_t be orthogonal in large samples, the $X_t's$ can be ignored in the analysis of the Wald statistic[5] which may be written

$$F = \widehat{\gamma}_0' \left(\widehat{\sigma}^{-2} \sum_{t=1}^{T} Z_t Z_t' \right) \widehat{\gamma}_0 \tag{3.4}$$

[5]The usual form of the F-statistic is our F multiplied by $(T - s + 1)/2T$.

where $\widehat{\gamma}_0$ is the OLS estimator of γ_0 and $\widehat{\sigma}^2 = T^{-1}\sum_{t=1}^{T} e_t^2$ is the estimator of σ_ε^2. The new statistic is

$$\underline{\omega} = \sum_{j=1}^{[s/2]} \underline{\omega}_j \tag{3.5}$$

where each $\underline{\omega}_j$, $j = 1,..,[s/2]$, is defined as in proposition 1.1 but using the residuals $\underline{e}_t$ and with the summation running in the reverse order, that is

$$\underline{\omega}_j = a_j T^{-2} \underline{\sigma}^{-2} \sum_{t=1}^{T} \left[\left(\sum_{i=t}^{T} \underline{e}_i \cos \lambda_j i \right)^2 + \left(\sum_{i=t}^{T} \underline{e}_i \sin \lambda_j i \right)^2 \right], \qquad j = 1,..,[s/2], \tag{3.6}$$

where $\underline{\sigma}_\varepsilon^2 = T^{-1}\sum_{t=1}^{T} \underline{e}_t^2$. Using the arguments of King and Hillier (1985) and Taylor (2002a) it is easy to show that, when W is specified as in the footnote below formula (1.6), $\underline{\omega}$ is the LBI test statistic[6] against H_1^S for the model in (1.1)-(1.4) with $\gamma_0 = 0$.

The following proposition provides the asymptotic distribution of F and $\underline{\omega}$ under the local alternative hypotheses $H_{1,T}^D$, $H_{1,T}^S$ where $H_{1,T}^D : \gamma_0 = c_D \iota/\sqrt{T}$, $\sigma_\kappa^2 = 0$ and $H_{1,T}^S : \gamma_0 = 0$, $\sigma_\kappa^2 = c_S^2/T^2$, where ι is a $s-1$ vector of ones and c_D, c_S are fixed constants. This provides the basis of a power comparison between the two tests.

Proposition 3.1. *Let y_t be generated by the model (1.1)-(1.2),(3.1)-(3.3) with $W = I_{s-1}$ and with the nonseasonal regressors X_t satisfying Assumption A1 of section 1. Let $W_{0,s-1}(r)$, $W_{1,s-1}(r)$ be independent standard Wiener processes of dimension $s-1$, and let $\Lambda = diag\,(1/2, ..., 1/2, 1)$ when s is even and $\Lambda = 1/2 I_{s-1}$ when s is odd. Then*
(i) under $H_{1,T}^D$,

$$F \xrightarrow{d} V_{D,s-1}(0; c_D)' V_{D,s-1}(0; c_D)$$
$$\underline{\omega} \xrightarrow{d} \int_0^1 V_{D,s-1}(r; c_D)' V_{D,s-1}(r; c_D) dr$$

[6] In the LBI statistic the summations run in reverse order, from t to T as opposed from 1 to t; in the tests of section 1 the reverse summations can be replaced by the more usual direct summations as fitting the seasonal regressors implies $\sum_{i=1}^{T} Z_i e_i = 0$.

where $V_{D,s-1}(r;c_D) = W_{0,s-1}(1-r) + c_D\sigma^{-1}\Lambda^{\frac{1}{2}}(1-r)\iota$, $r \in [0,1]$,
(ii) under $H^S_{1,T}$,

$$F \xrightarrow{d} V_{S,s-1}(0;c_S)'V_{S,s-1}(0;c_S)$$
$$\underline{\omega} \xrightarrow{d} \int_0^1 V_{S,s-1}(r;c_S)'V_{S,s-1}(r;c_S)dr$$

where $V_{S,s-1}(r;c_S) = W_{0,s-1}(1-r) + c_S\sigma^{-1}\Lambda^{\frac{1}{2}}\int_r^1 W_{1,s-1}(s)ds$, $r \in [0,1]$,
(iii) under either H^D_1 *or* H^S_1, F *and* $\underline{\omega}$ *are* $O_p(T)$.

Remark 1. *The asymptotic distribution of* F *under* $H^D_{1,T}$ *is a noncentral chi-square distribution with* $s-1$ *degrees of freedom and noncentrality parameter equal to* $c_D^2\iota'\Lambda\iota/\sigma^2$.

Remark 2. *Under both local alternatives, i.e. when* $\gamma_0 = c_D\iota/\sqrt{T}$ *and* $\sigma^2_\kappa = c_S^2/T^2$, *the asymptotic distribution of* F *and* $\underline{\omega}$ *are constructed using the process* $V(r;c_D,c_S) = W_{0,s-1}(1-r) + c_D\Lambda^{\frac{1}{2}}(1-r)\iota + c_S\Lambda^{\frac{1}{2}}\int_r^1 W_{1,s-1}(s)ds$, $r \in [0,1]$, *instead of either* $V_{D,s-1}(r;c_D)$ *or* $V_{S,s-1}(r;c_S)$.

Remark 3. *A modification of* $\underline{\omega}$ *would be to replace* $\underline{\sigma}^2$ *with* $\widehat{\sigma}^2$, *that is to fit seasonal dummies when calculating the denominator of the statistics in (3.6). This makes no difference to the asymptotic distribution under the null and the local alternative hypotheses.*

Although the asymptotic distribution of $\underline{\omega}$ under the null hypothesis differs from that of ω, it still belongs to the Cramer-von Mises family. The 5% critical values for one, two and three degrees of freedom - the last appropriate for a full test on quarterly data - are 1.65, 2.63 and 3.46 respectively; see table 1 in Nyblom (1989, p227). The 5% critical value for eleven degrees of freedom, kindly supplied by J. Nyblom, is 9.03. For the reasons given in section 4, the asymptotic distribution is unaffected by the inclusion of a constant or a constant and a time trend.

The asymptotic distributions of F and $\underline{\omega}$ under the local alternatives $H^D_{1,T}$, $H^S_{1,T}$ can be used to compare the power performance of the two tests. This is done in table 6 Specifically, for a quarterly model, $s=4$, we have generated 50000 replications of the limiting random variables defined in proposition 3.1 by replacing the continuous time Wiener processes $W_{0,s-1}$, $W_{1,s-1}$ by their discrete counterparts (dividing the unit interval into 1000 parts). We have also considered the limiting behavior of the ω test, invariant to the presence of deterministic seasonality; its asymptotic distribution against $H^S_{1,T}$ is given in Taylor (2002a). Note that under both the fixed and local alternative H^D_1, $H^D_{1,T}$ the asymptotic power of this test is equal to its nominal size.

Table 6 therefore reports, for a quarterly model, the local asymptotic power of the three tests, at the nominal 5% significance level across a range of values for the parameters c_D, c_S (with σ^2 set equal to 1). As expected, the Wald test is more powerful under the local alternative of deterministic seasonality, while $\underline{\omega}$ achieves the highest power under pure stochastic seasonality, being the LBI test for this case. For example for $c_D = 2$ and $c_S = 0$, the asymptotic power of the Wald test is 0.652, as opposed to 0.559 for the test based on $\underline{\omega}$. On the other hand, for $c_S = 4$ and $c_D = 0$, the power of the $\underline{\omega}$ test is 0.668 while that of the Wald test is 0.627. Note finally that, under pure stochastic seasonality, the power of the ω test of section 1 is considerably lower than that of $\underline{\omega}$.

The local power of the modified test, using $\widehat{\sigma}^2$ rather than $\underline{\sigma}^2$, is, as suggested in Remark 3, the same as that of $\underline{\omega}$. However, in practice, it may well have a higher power than $\underline{\omega}$ against deterministic seasonality. This is because when $\sigma^2_\kappa = 0$ the probability limit of $\underline{\sigma}^2$ exceeds that of $\widehat{\sigma}^2$; indeed since $\underline{\sigma}^2 \geq \widehat{\sigma}^2$ the modified statistic will always be greater than or equal to $\underline{\omega}$.[7]

A second modification is also in order. As formulated in (3.6), the test is LBI against stochastic seasonality with $\gamma_0 = 0$. In practice, we are more concerned with seasonal patterns that diminish over time. Thus our recommendation is to use the *forward* summation just as in the ω test of section 1 since this would be the LBI test if the data were generated backwards starting with $\gamma_{T+1} = 0$. Taking these points into consideration our preferred statistic, $\underline{\underline{\omega}}$, is constructed using

$$\underline{\underline{\omega}}_j = a_j T^{-2}\widehat{\sigma}^{-2}\sum_{t=1}^{T}\left[\left(\sum_{i=1}^{t}\underline{e}_i\cos\lambda_j i\right)^2 + \left(\sum_{i=1}^{t}\underline{e}_i\sin\lambda_j i\right)^2\right], \qquad j = 1,..,[s/2]. \tag{3.7}$$

When ε_t is serially correlated, the $\underline{\underline{\omega}}$ test can be modified as in sub-section 1.3. If the spectrum is computed using the residuals after fitting the seasonal regressors the statistic will be denoted $\underline{\underline{\omega}}^*(m)$. The test can be extended to deal with both serial correlation and heteroscedasticity by making the amendment of (1.9).

[7] There is a parallel with the test on $\widehat{\gamma}_0$ in that using $\underline{\sigma}^2$ instead of $\widehat{\sigma}^2$ would give the LM statistic.

The Wald test can be carried out by fitting a model that is unrestricted except insofar as the seasonal component is taken to be nonstochastic, that is σ_κ^2 is set to zero. Alternatively a nonparametric test can be set up using a nonparametric covariance matrix estimator as in Andrews (1991). This is essentially the same correction as in (1.9). To be specific

$$F(m) = T\widehat{\gamma}_0'[Q^{-1}\widehat{\Omega}(m)Q^{-1}]^{-1}\widehat{\gamma}_0, \tag{3.8}$$

where $Q = T^{-1}\sum_{t=1}^{T} Z_t Z_t'$. If there is no need to guard against heteroscedasticity, the modifications are simply made using estimates of the spectrum as for $\underline{\omega}^*(m)$.

3.2. Example: A Diminishing Seasonal Pattern

As an example we consider the logarithm of 3-month money market interest rate in Spain for the period 1977Q1-2001Q4; the source is the Bank of International Settlements (BIS) macroeconomic series database. The series is depicted in figure 2a. It is difficult to detect a seasonal pattern from a casual glance at the graph and one would not normally expect one to be present in an interest rate series, the functioning of the interbank loans market may imply some seasonality, as in Hamilton (1996).

Fitting the BSM[8] to the series gives a seasonal component[9] as shown in figure 2. The chi-square statistic for the seasonals at the end of the series is only 0.09 which is clearly not significant as the 5% critical value for a χ_3^2 is 7.81. However the graph shows a fairly strong seasonal pattern until the mid-eighties. The question is whether the pattern as a whole is in any sense significant.

Setting the seasonal variance to zero and re-estimating the BSM gives a Wald statistic of 4.76, with a p-value of 0.19. This is still not significant. If the series is differenced and a nonparametric Wald test, (3.8), is computed using the Newey-West covariance matrix estimator with three lags a similar p-value, 0.17, is obtained. On the other hand, the spectral nonparametric statistic computed using forward summations takes the values 3.83 and 3.01 for $m = 3$ and 6 respectively, rising to 4.64 and 3.89 for $\underline{\underline{\omega}}^*(m)$, the preferred form in which the spectrum is estimated after fitting seasonal regressors. As the 5% critical value is 3.46, this test provides a firm rejection of the hypothesis of an insignificant seasonal pattern.

[8] The slope variance is estimated to be zero and the estimate of the (fixed) slope is small and insignificant. Removing the slope altogether makes little difference to the results.

[9] If the raw series is used, the seasonal pattern is similar. We have used logarithms as the diagnostics are better.

Finally, for $m = 3$ and 6 the $\omega(m)$ statistic of section 1 takes the values 1.17 and 1.02 respectively (against a 5% critical value of 1.00); thereby confirming the presence of stochastic seasonality.

3.3. Seasonal Adjustment

The above tests can be applied to a seasonally adjusted series to see if the adjustment has been effective. This assumes that the adjustment has been done by means of moving averages, rather than by regressing on seasonal dummies. If dummies have been used, then the ω test statistics have the asymptotic distributions of section 1.

3.4. Detection of Trading Day Effects

Cleveland and Devlin (1980) showed that peaks at certain frequencies in the estimated spectra of monthly time series indicate the presence of trading day effects. Specifically there is a peak at a frequency of $0.348 \times 2\pi$ radians, with the possibility of subsidiary peaks at $0.432 \times 2\pi$ and $0.304 \times 2\pi$ radians. An option in the output of the X-12-ARIMA program provides a comparison of the estimates of these frequencies with the adjacent frequencies; see Soukup and Findley (2000). However, there is no formal test. One possibility is to construct parametric or nonparametric statistics analogous to $\underline{\omega}_j$, $\underline{\underline{\omega}}_j$ so as to carry out tests for permanent cyclical effects at one or all of the three trading day frequencies. Assuming that no (deterministic) trading day model has been fitted, the asymptotic distributions under the null will be CvM_0, with the 5% critical value being 2.63 for a test at a single frequency and 5.68 for a test at all three frequencies.

As an example we took the irregular component, obtained from X12-ARIMA, of series s0b56ym, *U.S. Retail Sales of Children's, Family, and Miscellaneous Apparel,* as supplied by the Bureau of the Census. Since the process followed by this irregular component cannot be derived, it was decided to use the nonparametric test. The test statistic with ten lags for the single main frequency was 7.03. For all three frequencies it was 8.21. Both give a clear rejection of the null hypothesis that there is no trading day effect.

4. Conclusion

The seasonality test statistic proposed by Canova and Hansen may be simplified so that a nonparametric correction for serial correlation is based on estimating the spectrum of the series at the relevant seasonal frequency or frequencies. This test statistic then has a very straightforward interpretation. As might be expected, Monte Carlo experiments show a slight gain in power over the original Canova-Hansen test for homoscedastic series, but a size distortion and lower power when there is seasonal heteroscedasticity.

If a model is fitted, a parametric seasonality test may be based on the innovations or smoothing errors, but Monte Carlo experiments show that they have similar properties. If the main reason for fitting a model is to investigate seasonality, a basic structural time series model, consisting of stochastic trend, seasonal and irregular components, will usually be adequate. However, it is worth noting that the innovations test can be implemented for any structural time series model, including those that do not have a time invariant structure. Nonparametric tests require a decision about lag truncation, but our Monte Carlo experiments show that, in samples of size 200, the rejection probabilities do not fall by very much when the lag length is increased from four to eight. Nonparametric tests are also dependent on decisions regarding differencing, but an important practical finding to emerge from the Monte Carlo experiments is that for most economic time series taking first differences is likely to be a good strategy.

The Monte Carlo experiments indicate higher probabilities of rejection from parametric tests, but any assessment of power has to be offset against the higher size. For quarterly data, there is a tendency for parametric tests to be oversized at frequency π and this carries over to the joint test. The actual sizes of the joint tests at the 5% level of significance in a sample of size 200 are nearly 10%. On the other hand, with a lag length of four the actual size of the corresponding nonparametric tests never exceeded 5.5%. Parametric tests are attractive within the context of a model-building exercise, but if the sole focus is on testing for stochastic seasonality, there is no overwhelming case for using them in preference to nonparametric tests.

If there are breaks in the seasonal pattern, the seasonality test may be modified so as to have an asymptotic distribution that is independent of the position of the break points under the null hypothesis that the seasonal pattern is deterministic. The UK marriages example yields much greater values for the parametric test statistics both with and without the seasonal break dummy variables. In the modeled break case, the parametric test indicates a rejection of the null hypothesis while the conclusions from the nonparametric tests are ambivalent.

Although a fixed seasonal component is normally estimated under the null hypothesis, there may be situations where the researcher wishes to carry out a general test against a permanent seasonal component, irrespective of whether it be deterministic and stochastic. We propose the use of test statistics that have the same form as the tests against nonstationary seasonality except insofar as no fixed seasonal effects are removed when the residuals used to construct the partial sums are formed. The asymptotic critical values are easily obtained. We compared the test with a Wald test carried out to determine the joint significance of a set of seasonal dummies assumed to be constant over time. This test can also be carried out nonparametrically. When only deterministic seasonality is present, an analysis of local power shows the Wald test to be more powerful than our modification of the test against nonstationary seasonality, but not by very much. When the seasonal pattern is diminishing over time, something that occurs frequently with macroeconomic series, the modified test against nonstationary seasonality is slightly more attractive. However, in the example on Spanish interest rates this test shows a clear rejection of the null hypothesis of no permanent seasonality whereas the Wald test does not reject. Statistics constructed in a similar way can be used to test against trading day effects by exploiting the fact that these give rise to cycles at known frequencies.

An appealing feature of the proposed test statistics is that under the null hypothesis they all have asymptotic distributions belonging to the Cramér-von Mises family. Thus they provide an integrated approach to testing a wide range of hypotheses that arise in the context of seasonal time series.

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APPENDICES

A. Asymptotic distribution

Proof of Proposition 1.1

From assumptions A1-A2 we have that, under H_0, $T^{\frac{1}{2}}D_T(\widehat{\beta}-\beta)$ and $T^{\frac{1}{2}}(\widehat{\gamma}_0-\gamma_0)$

are $O_p(1)$ and asymptotically orthogonal. In particular,

$$T^{\frac{1}{2}}\left(\widehat{\gamma}_0-\gamma_0\right) \xrightarrow{d} N\left(0, G^{-1}\right), \tag{A.1}$$

where G is a $(s-1)\times(s-1)$ diagonal matrix whose elements are proportional to the spectral generating function evaluated at the seasonal frequencies λ_j, $j = 1, ..., [s/2]$; when s is even

$$G = \mathrm{diag}\left(\frac{1}{2}g(\lambda_1), \frac{1}{2}g(\lambda_1), ..., \frac{1}{2}g(\lambda_{s/2-1}), \frac{1}{2}g(\lambda_{s/2-1}), g(\lambda_{s/2})\right),$$

while, when s is odd, the last two diagonal elements of G are both equal to $\frac{1}{2}g(\lambda_{[s/2]})$.

Note that asymptotic orthogonality is a direct consequence of assumption A2 (ii), whereas the result (A.1) mainly follows from the Central Limit Theorem of Brillinger (1975, theorem 4.4.1), namely that for $j = 1, ..., s^*$

$$\begin{pmatrix} T^{-\frac{1}{2}}\sum_{t=1}^{T}\varepsilon_t\cos\lambda_j t \\ T^{-\frac{1}{2}}\sum_{t=1}^{T}\varepsilon_t\sin\lambda_j t \end{pmatrix} \xrightarrow{d} N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{2}g(\lambda_j) & 0 \\ 0 & \frac{1}{2}g(\lambda_j) \end{pmatrix}\right), \tag{A.2}$$

while, when s is even, $T^{-\frac{1}{2}}\sum_{t=1}^{T}\varepsilon_t\cos\lambda_{s/2}t \xrightarrow{d} N\left(0, g(\lambda_{s/2})\right)$; in addition, the limiting random vectors of (A.2) above are also independent across j. Furthermore a Functional Central Limit Theorem also holds, i.e. the partial sums of $\varepsilon_t\cos\lambda_j t$ and $\varepsilon_t\sin\lambda_j t$ weakly converge to independent Wiener processes; see Chan and Wei (1988).

Now write the OLS residuals as

$$e_t = \varepsilon_t - X_t'(\widehat{\beta}-\beta) - Z_t'(\widehat{\gamma}_0-\gamma_0), \qquad t = 1, ..., T,$$

and, for $j = 1, ..., [s/2]$, consider the (normalised) partial sum process $S_{j,T}(r) =$

$T^{-\frac{1}{2}}\sum_{t=1}^{[Tr]} z_{jt}e_t$, $r \in [0,1]$. Then we have that, under $\mathrm{H_0}$,

$$\begin{aligned} S_{j,T}(r) &= T^{-\frac{1}{2}}\sum_{t=1}^{[Tr]} z_{jt}\varepsilon_t - T^{-1}\sum_{t=1}^{[Tr]} z_{jt}X_t' D_T^{-1} D_T T^{\frac{1}{2}}(\widehat{\beta}-\beta) \\ &\quad -T^{-1}\sum_{t=1}^{[Tr]} z_{jt}Z_t' T^{\frac{1}{2}}(\widehat{\gamma}_0-\gamma_0) \\ &= T^{-\frac{1}{2}}\sum_{t=1}^{[Tr]} z_{jt}\varepsilon_t - T^{-1}\sum_{t=1}^{[Tr]} z_{jt}Z_t'\left(T^{-1}\sum_{t=1}^{T} Z_t Z_t'\right)^{-1} T^{-\frac{1}{2}}\sum_{t=1}^{T} Z_t\varepsilon_t + o_p(1) \\ &= T^{-\frac{1}{2}}\sum_{t=1}^{[Tr]} z_{jt}\varepsilon_t - rT^{-\frac{1}{2}}\sum_{t=1}^{T} z_{jt}\varepsilon_t + o_p(1), \end{aligned}$$

where the last expression is due to the orthogonality relation $\lim_{T\to\infty} T^{-1}\sum_{t=1}^{T} z_{jt}z_{ht} = 0$ for $j \neq h$.

Then using the functional central limit theorem of Chan and Wei (1988), and the continuous mapping theorem, we have that, under H_0,

$$(a_j g(\lambda_j))^{-\frac{1}{2}} S_{j,T}(r) \Rightarrow B_{a_j}(r), \quad r \in [0,1], \tag{A.3}$$

where $B_k(r) = W_k(r) - rW_k(1)$ is a k-dimensional standard Brownian bridge, $W_k(r)$ is a k-dimensional standard standard Wiener process and $\Rightarrow$ denotes weak convergence; furthermore $S_{j,T}(r)$ is asymptotically independent of $S_{h,T}(r)$ for $j \neq h$. As under $\mathrm{H_0}$ and by assumption A3 $\widehat{g}(\lambda_j;m) \xrightarrow{p} g(\lambda_j)$, it then follows by the continuous mapping theorem that the null limiting distributions of $\omega_j(m)$ when $A = A_j$, $j = 1, ..., [s/2]$, and $\omega(m)$ when $A = \mathrm{I}_{s-1}$ are $CvM(a_j)$ and $CvM(s-1)$ respectively.

Under $\mathrm{H_1} : \sigma_\kappa^2 > 0$, when $A = A_j$ it is easily seen that the partial sum $S_{j,T}(r)$ is $O_p(T^{\frac{1}{2}})$ while assumption A3 implies that $\widehat{g}(\lambda_j;m)$ is $O_p(m)$, see Stock (1994, p. 2797-2799). Thus both $\omega_j(m)$ and $\omega(m)$ are $O_p(T/m)$.

Proof of Proposition 2.1

Consider the two subsamples $\{1, ..., [\alpha T]\}$, $\{[\alpha T]+1, ..., T\}$, and let $S_{j,T}^A(r) = [\alpha T]^{-\frac{1}{2}}\sum_{t=1}^{[\alpha Tr]} z_{jt}e_t$, $r \in [0,1]$, and $S_{j,T}^B(r) = [(1-\alpha)T]^{-\frac{1}{2}}\sum_{t=[\alpha T]+1}^{[Tr]} z_{jt}e_t$, $r \in [0,1]$, be the partial sum processes in the first and second subsample respectively. Using a functional central limit theorem as in (A.3), we have that, under $\mathrm{H_0}$,

$$(a_j g(\lambda_j))^{-\frac{1}{2}}\left(S_{j,T}^A(r)', S_{j,T}^B(r)'\right)' \Rightarrow \left(B_{a_j}^A(r)', B_{a_j}^B(r)'\right)', \quad r \in [0,1],$$

where $B^A_{a_j}(r)$, $B^B_{a_j}(r)$ are independent k-dimensional standard Brownian bridges. Noticing that

$$\sum_{t=1}^{T} c_{jt}k_t = [\alpha T]^{-1} \sum_{t=1}^{[\alpha T]} trace\left(S^A_{j,T}\left(\frac{t}{T}\right) S^A_{j,T}\left(\frac{t}{T}\right)'\right)$$
$$+[(1-\alpha)T]^{-1} \sum_{t=[\alpha T]+1}^{T} trace\left(S^A_{j,T}\left(\frac{t}{T}\right) S^A_{j,T}\left(\frac{t}{T}\right)'\right),$$

it then follows by the functional central limit theorem of Chan and Wei (1988), the continuous mapping theorem and the additivity property of independent Cramer-von Mises random variables that $\omega^*_j(\alpha;m) \xrightarrow{d} CvM(2a_j)$ and $\omega^*(\alpha;m) \xrightarrow{d} CvM(2s-2)$ under H_0. Then, from the same arguments of proposition 2.1, under $\mathrm{H}_1 : \sigma^2_\kappa > 0$, when $A = A_j$ both $\omega^*_j(\alpha;m)$ and $\omega^*(\alpha;m)$ are $O_p(T/m)$.

B. LBI test

When ε_t in (1.1) is generalised so as to be a linear stationary process, possibly consisting of more than one component, its $T \times T$ matrix covariance matrix will be denoted $V = \sigma^2_* V_*$, where σ^2_* is a variance parameter. If V_* is known, it follows from King and Hillier (1985), that the LBI test is of the form (1.5) the OLS residuals are replaced by the elements of the $T \times 1$ vector, $V^{-1}_* \widetilde{e}$, where $\widetilde{e}$ is the $T \times 1$ vector of generalised least squares (GLS) residuals. The LBI test against stochastic seasonality at all frequencies is similarly constructed. If ε_t contains a white noise component with variance σ^2_* then it is straightforward to show that $V^{-1}_* \widetilde{e}$ is equal to the smoothed estimator of the vector of the white noise series. More generally, when multiplied by σ^{-2}_* it becomes the $T \times 1$ vector of smoothing errors, denoted $u = V^{-1}\widetilde{e}$. The smoothing errors are produced as a by-product of the smoother applied to the state space form of the model; see de Jong and Penzer (1998) and Harvey and Streibel (1997).

With V_* known, an exact test can be carried out using numerical inversion to construct critical values or probability values. However, V_* will normally depend on unknown parameters, so there are good reasons for wishing to use a statistic with a known asymptotic distribution. If the test statistic is formed from smoothing errors, it is necessary to take account of their serial correlation. Following a similar argument to the one used to give (1.7), the denominator needs an estimator of the spectral generating function of $V^{-1}\varepsilon$. This is equal to $1/g_\varepsilon(\lambda)$, where

$g_\varepsilon(\lambda)$ is the spectral generating function of ε_t. The parametric test statistic is therefore

$$\omega_j = a_j T^{-2}\widetilde{g}_\varepsilon(\lambda_j)\sum_{t=1}^{T}\left[\left(\sum_{i=1}^{t} u_i \cos\lambda_j i\right)^2 + \left(\sum_{i=1}^{t} u_i \sin\lambda_j i\right)^2\right], \qquad j = 1, ..., [s/2], \tag{B.1}$$

where u_i is the $i-th$ smoothing error. The test statistic has the same asymptotic distribution as (1.5), namely $CvM(2)$. This remains true when parameters in V_* are estimated; compare Leybourne and McCabe (1994) and Saikkonen and Luukkonen (1993).

The above correction can be made even if the model contains a stochastic trend. The smoothing error series is stationary and though it is not (strictly) invertible, the noninvertibility only affects the zero frequency and the 'quasi' sgf of the non-seasonal part of the model can be inverted at λ_j. Thus for the special case of (1.11) in which the trend is a random walk, $\widetilde{g}_\varepsilon(\lambda_j)$ in (B.1) is replaced by

$$\left[\frac{\widetilde{\sigma}_\eta^2 + 2(1-\cos\lambda)\widetilde{\sigma}_\varepsilon^2}{2(1-\cos\lambda)}\right], \quad 0 < \lambda \leq \pi \tag{B.2}$$

If instead of the smoothing errors the smoothed estimator of an irregular component, ε_t, is used, the above correction factor must be divided by $\widetilde{\sigma}_\varepsilon^4$.

C. Asymptotics for tests against permanent seasonality

To prove Proposition 3.1 we need the following lemma.

Lemma C.1. *(i) Under* $H_{1,T}^S : \gamma_0 = 0,\ \sigma_\kappa^2 = c_S^2/T^2 > 0$

$$T^{-\frac{1}{2}}\sum_{t=[Tr]}^{T} Z_t Z_t' \sum_{i=1}^{t}\kappa_i \Rightarrow c_S\Lambda\int_r^1 W_{1,s-1}(r)dr, \quad r\in[0,1].$$

(ii) Under $H_1^S : \gamma_0 = 0,\ \sigma_\kappa^2 > 0$

$$T^{-1}\sum_{t=1}^{T} D_T^{-1}X_t Z_t' \sum_{i=1}^{t}\kappa_i \xrightarrow{p} 0.$$

Proof of the Lemma.

To avoid unnecessary complications in the notations we assume that $T_s \equiv T/s$ is an integer. Let $t^* = (\bar{t}-1)/s+1$, where $\bar{t} = 1, ..., T$. For part (i), first note that

$$\sum_{t=\bar{t}}^{T}\sum_{i=1}^{t}\kappa_i = \begin{cases} \sum\limits_{j=1}^{s}\sum\limits_{u=t^*}^{T_s}(B_u + R_{j,u}) & \text{if } t^* \text{ is an integer} \\ \sum\limits_{j=1}^{s}\sum\limits_{u=[t^*]}^{T_s}(B_u + R_{j,u}) - \sum\limits_{i=[t^*]}^{\bar{t}-1}\kappa_i & \text{otherwise,} \end{cases}$$

where, for $u = 1, ..., T_s$,

$$B_u = \begin{cases} 0 & u = 1 \\ \sum_{i=1}^{s(u-1)}\kappa_i & u > 1 \end{cases}$$

$$R_{j,u} = \sum_{i=s(u-1)+1}^{s(u-1)+j}\kappa_i, \quad j = 1, ..., s.$$

Note that we can write, for $t = 1, ..., T$,

$$Z_t = \begin{cases} Z_j & \text{if } 1 \leq j \leq s-1 \\ Z_s & \text{if } j = 0 \end{cases}$$

where $j = t \bmod s$. Then, using the decomposition above, we have that under the local alternative $H_{1,T}^S$, for $r \in [0,1]$,

$$\begin{aligned} T^{-\frac{1}{2}}\sum_{t=[Tr]}^{T} Z_t Z_t' \sum_{i=1}^{t}\kappa_i &= \sum_{j=1}^{s}\frac{1}{sT_s}\sum_{u=[T_s r]}^{T_s} Z_j Z_j' T^{\frac{1}{2}}(B_u + R_{j,u}) + o_p(1) \\ &= \sum_{j=1}^{s}\frac{Z_j Z_j'}{s}\frac{1}{T_s}\sum_{u=[T_s r]}^{T_s} T^{\frac{1}{2}}(B_u + R_{j,u}) + o_p(1). \end{aligned}$$

By a standard functional central limit theorem we obtain that, under $H_{1,T}^S$,

$$T^{\frac{1}{2}}B_{[T_s r]} \Rightarrow c_S W_{1,s-1}(r), \quad r \in [0,1],$$

with $W_{1,s-1}(r)$ being an $s-1$ dimensional standard Wiener process, while, for all $j = 1, ..., s$,

$$T^{-\frac{1}{2}}\sum_{u=[T_s r]}^{T_s} R_{j,u} \xrightarrow{p} 0.$$

Thus, by an application of the Continuous Mapping Theorem, for $r \in [0,1]$,

$$T^{-\frac{1}{2}} \sum_{t=[Tr]}^{T} Z_t Z_t' \sum_{i=1}^{t} \kappa_i \xrightarrow{d} c_S \Lambda \int_r^1 W_{1,s-1}(r) dr,$$

where $\Lambda = s^{-1} \sum_{j=1}^{s} Z_j Z_j'$ is the diagonal matrix defined in the statement of the proposition.

For part (ii) first note that

$$\sum_{j=1}^{s} Z_j' = 0. \tag{C.1}$$

Then proceeding in a similar way as above we have that

$$\frac{1}{T} \sum_{t=1}^{T} D_T^{-1} X_t Z_t' \sum_{i=1}^{t} \kappa_i = \sum_{j=1}^{s} \frac{1}{sT_s} \sum_{u=1}^{T_s} D_T^{-1} X_{su} Z_j' (B_u + R_{j,u}) + o_p(1) \tag{C.2}$$

$$= \frac{1}{sT_s} \sum_{u=1}^{T_s} D_T^{-1} X_{su} \widetilde{\kappa}_u + o_p(1) \tag{C.3}$$

where the second equality uses (C.1) and

$$\widetilde{\kappa}_u \equiv \sum_{j=1}^{s} Z_j' R_{j,u}.$$

Now $\widetilde{\kappa}_u$, $u = 1, ..., T_s$ is an independent sequence as each element is made as weighted sum of s nonoverlapping disturbances κ_t, $t = 1, ..., T$. Part (ii) of the lemma is therefore proved by applying to (C.3) a Law of Large Number for independent heteroscedastic sequences.

Proof of proposition 3.1

Let y_t be generated by the model (1.1)-(1.2),(3.1)-(3.3) with $W = I_{s-1}$ and let $\widehat{\beta}$, $\widehat{\gamma}_0$ be the OLS estimators of β, γ_0 from regressing y_t on $(X_t', Z_t')'$ and $\underline{\beta}$ be the OLS estimator from regressing y_t on X_t. Then

$$\begin{pmatrix} \widehat{\beta} - \beta \\ \widehat{\gamma}_0 - \gamma_0 \end{pmatrix} = A^{-1} b$$

where

$$A = \begin{pmatrix} T^{-1}\sum_{t=1}^{T} D_T^{-1}X_tX_t'D_T^{-1} & T^{-1}\sum_{t=1}^{T} D_T^{-1}X_tZ_t' \\ T^{-1}\sum_{t=1}^{T} Z_tX_t'D_T^{-1} & T^{-1}\sum_{t=1}^{T} Z_tZ_t' \end{pmatrix},$$

$$b = \begin{pmatrix} T^{-1}\sum_{t=1}^{T} D_T^{-1}X_t\left(\varepsilon_t + Z_t'\sum_{i=1}^{t}\kappa_i\right) \\ T^{-1}\sum_{t=1}^{T} Z_t\left(\varepsilon_t + Z_t'\sum_{i=1}^{t}\kappa_i\right) \end{pmatrix},$$

and

$$\left(\underline{\beta} - \beta\right) = \left(T^{-1}\sum_{t=1}^{T} D_T^{-1}X_tX_t'D_T^{-1}\right)^{-1} T^{-1}\sum_{t=1}^{T} D_T^{-1}X_t\left(\varepsilon_t + Z_t'\sum_{i=1}^{t}\kappa_i + Z_t'\gamma_0\right).$$

The OLS residuals for the two regressions can be written respectively as

$$e_t = \varepsilon_t + Z_t'\sum_{i=1}^{t}\kappa_i - X_t'\left(\widehat{\beta} - \beta\right) - Z_t'\left(\widehat{\gamma}_0 - \gamma_0\right) \tag{C.4}$$

and

$$\underline{e}_t = \varepsilon_t + Z_t'\sum_{i=1}^{t}\kappa_i + Z_t'\gamma_0 - X_t'\left(\underline{\beta} - \beta\right). \tag{C.5}$$

By the orthogonality of the regressors Z_t, X_t of assumption A1, it can be straightforwardly shown that $\underline{\beta} = \widehat{\beta} + o_p(1)$. Furthermore, by the Functional Central Limit Theorem of Chan and Wei (1988) we have the following weak convergence result

$$\sigma^{-1}\Lambda^{-\frac{1}{2}}T^{-\frac{1}{2}}\sum_{t=[Tr]}^{T} Z_t\varepsilon_t \Rightarrow W_{\mathbf{0},s-1}(1-r), \quad r \in [0,1], \tag{C.6}$$

where $W_{\mathbf{0},s-1}(r)$ and Λ are defined in the statement of the proposition. Note that

$$\lim_{T\to\infty} T^{-1}\sum_{t=1}^{T} Z_tZ_t' = \Lambda.$$

Consider the local alternative $H_{1,T}^{D}: \gamma_0 = c_D\iota/\sqrt{T}$, $\sigma_\kappa^2 = 0$. We have that $\widehat{\beta} = \beta + o_p(1)$,

$$\sigma^{-1}\Lambda^{\frac{1}{2}}T^{\frac{1}{2}}\widehat{\gamma}_0 \xrightarrow{d} W_{\mathbf{0},s-1}(1) + c_D\sigma^{-1}\Lambda^{\frac{1}{2}}\iota = V_{D,s-1}(0;c_D),$$

and $e_t = \varepsilon_t + o_p(1)$. Then $F \xrightarrow{d} V_{D,s-1}(0;c_D)'V_{D,s-1}(0;c_D)$; note that, as $W_{0,s-1}(1)$ is a standard $s-1$ dimensional Normal, the asymptotic distribution of F is a non-central chi-square distribution with $s-1$ degrees of freedom and noncentrality parameter equal to $c_D^2\sigma^{-2}\iota'\Lambda\iota$.

Furthermore, from (C.6) and (C.5),

$$\sigma^{-1}\Lambda^{-\frac{1}{2}}T^{-\frac{1}{2}}\sum_{t=[Tr]}^{T} Z_t\underline{e}_t \Rightarrow W_{0,s-1}(1-r) + c_D\sigma^{-1}\Lambda^{\frac{1}{2}}(1-r)\iota = V_{D,s-1}(r;c_D), \;\; r\in[0,1]$$

and since $\underline{e}_t = \varepsilon_t + o_p(1)$, $\underline{\omega} \xrightarrow{d} \int_0^1 V_{D,s-1}(r;c_D)'V_{D,s-1}(r;c_D)dr$.

Under the fixed alternative H_1^D, using similar arguments it is not difficult to show that $\widehat{\gamma}_0$ is $O_p(1)$, $\widehat{\sigma}^2$ and $\underline{\sigma}^2$ are $O_p(1)$, $\sum_{i=t}^{T} Z_t\underline{e}_t$ is $O_p(T)$. Thus both F and $\underline{\omega}$ are $O_p(T)$.

Consider the local alternative $H_{1,T}^S$. By (C.6) and using part (i) of the lemma we have that

$$\sigma^{-1}\Lambda^{\frac{1}{2}}T^{\frac{1}{2}}\widehat{\gamma}_0 \xrightarrow{d} W_{0,s-1}(1) + c_S\sigma^{-1}\Lambda^{\frac{1}{2}}\int_0^1 W_{1,s-1}(r)dr = V_{S,s-1}(0;c_S).$$

As, under $H_{1,T}^S$, $e_t = \varepsilon_t + o_p(1)$, it easily follows that $F \xrightarrow{d} V_{S,s-1}(0;c_S)'V_{S,s-1}(0;c_S)$.

Using similar arguments we also have that

$$\sigma^{-1}\Lambda^{-\frac{1}{2}}T^{-\frac{1}{2}}\sum_{t=[Tr]}^{T} Z_t\underline{e}_t \Rightarrow W_{0,s-1}(1-r) + c_S\sigma^{-1}\Lambda^{\frac{1}{2}}\int_r^1 W_{1,s-1}(s)ds = V_{S,s-1}(r;c_S), \;\; r\in[0,1],$$

and $\underline{\sigma}^2 = T^{-1}\sum_{t=1}^{T}\underline{e}_t^2 \xrightarrow{p} \sigma^2$. Thus

$$\begin{aligned}
\underline{\omega} &= \sum_{j=1}^{[s/2]} \underline{\omega}_j \\
&= T^{-2}\underline{\sigma}^{-2}\left(\sum_{t=1}^{T} Trace\left(\sum_{i=t}^{T} Z_i e_i\right)\left(\sum_{i=t}^{T} Z_i e_i\right)'\right) \\
&\xrightarrow{d} \int_0^1 V_{S,s-1}(r;c_S)'V_{S,s-1}(r;c_S)dr.
\end{aligned}$$

Finally, under the fixed alternative H_1^S, from part (ii) of the lemma we first obtain that both $\widehat{\beta} - \beta$ and $\underline{\beta}$-β are $o_p(1)$. Then, as $\sum_{i=t}^{T} Z_t \underline{e}_t$ is $O_p\left(T^{\frac{3}{2}}\right)$, $\widehat{\gamma}_0$ is $O_p\left(T^{\frac{1}{2}}\right)$, $\widehat{\sigma}^2$ and $\underline{\sigma}^2$ are $O_p(1)$, it easily follows that both the Wald statistic F and the statistic $\underline{\omega}$ are $O_p(T)$.

FIGURE HEADINGS
Figure 1 Trend and seasonals in UK marriages
Figure 2 Trend and seasonal in the logarithm of Spanish interest rates
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		Parametric tests						Nonparametric tests							
		Model known		Model estimated		BSM estimated		Spectral: levels		Spectral: FD		CH: levels		CH: FD	
	$\sigma_\kappa/\sigma_\varepsilon$	Innov.	Smooth.	Innov.	Smooth.	Innov.	Smooth.	m=4	m=8	m=4	m=8	m=4	m=8	m=4	m=8
$\lambda=\pi/2$	0	5.10	4.82	5.63	5.85	5.56	5.42	4.29	4.30	4.75	4.47	3.84	3.24	4.16	3.49
	0.010	8.52	8.10	9.36	9.97	9.23	9.90	7.31	7.23	7.77	7.31	6.32	5.49	6.74	5.61
	0.025	26.78	27.50	31.52	33.47	31.23	33.19	24.75	23.93	25.33	24.36	22.52	20.24	23.29	20.42
	0.050	60.37	64.35	74.13	77.96	74.07	77.78	58.80	56.68	58.78	56.56	56.36	52.43	56.44	52.01
	0.100	86.27	92.61	95.54	98.31	95.40	98.15	86.92	83.86	85.95	83.27	85.49	81.42	84.54	80.45
	0.500	96.75	100.00	98.70	100.00	98.70	100.00	98.70	96.71	97.72	95.89	98.41	95.50	97.21	94.37
$\lambda=\pi$	0	5.13	4.89	8.58	8.83	6.92	6.96	4.53	4.46	6.46	5.35	4.53	4.46	6.46	5.35
	0.010	10.01	9.53	14.92	15.26	14.67	15.04	8.61	8.35	11.24	9.71	8.61	8.35	11.24	9.71
	0.025	30.44	30.77	40.51	41.64	40.33	41.44	27.82	26.54	31.49	28.10	27.82	26.54	31.49	28.10
	0.050	58.05	60.71	74.66	77.40	74.28	77.04	54.48	50.80	57.46	52.47	54.48	50.80	57.46	52.47
	0.100	80.64	86.91	91.25	96.00	91.09	95.79	76.73	69.92	78.89	71.23	76.73	69.92	78.89	71.23
	0.500	94.05	100.00	96.43	100.00	96.50	100.00	92.12	83.20	93.08	83.84	92.12	83.20	93.08	83.84
Joint	0	5.01	4.60	9.58	9.79	7.00	6.91	3.98	3.95	5.28	4.55	3.11	2.45	4.29	2.83
	0.010	10.32	10.24	15.76	16.23	15.59	16.05	8.61	8.14	10.63	9.28	7.13	5.27	8.93	6.02
	0.025	40.93	41.66	47.25	48.18	46.99	47.90	36.71	34.61	40.00	36.52	32.84	26.80	35.94	28.88
	0.050	81.31	82.98	87.19	87.78	86.86	87.38	77.68	74.84	79.27	75.70	75.07	68.70	76.94	69.47
	0.100	97.76	98.77	99.46	99.52	99.29	99.34	96.32	94.47	96.63	94.63	95.56	92.11	95.85	92.25
	0.500	100.00	100.00	100.00	100.00	100.00	100.00	99.95	99.53	99.93	99.48	99.96	99.13	99.93	99.07

Table 1. Rejection frequencies for a random walk drift plus noise model with $\sigma_\eta/\sigma_\varepsilon$=0.1.

		Parametric tests						Nonparametric tests							
		Model known		Model estimated		BSM estimated		Spectral: levels		Spectral: FD		CH: levels		CH: FD	
	$\sigma_\kappa/\sigma_\varepsilon$	Innov.	Smooth.	Innov.	Smooth.	Innov.	Smooth.	m=4	m=8	m=4	m=8	m=4	m=8	m=4	m=8
$\lambda=\pi/2$	0	5.34	4.90	5.54	5.98	5.42	5.71	1.06	1.92	4.77	4.63	0.86	1.24	4.23	3.42
	0.010	7.73	7.35	8.74	9.28	8.58	9.13	2.01	3.21	7.16	6.86	1.63	2.29	6.35	5.28
	0.025	24.57	24.96	28.68	30.49	28.50	30.20	10.41	13.56	23.13	22.03	8.98	10.37	21.35	18.63
	0.050	57.16	60.63	70.62	74.25	70.46	73.99	39.18	43.43	55.88	54.00	36.51	38.30	53.47	49.22
	0.100	84.46	90.98	94.42	97.66	94.37	97.64	76.24	76.88	84.41	81.60	74.37	73.24	82.81	78.89
	0.500	96.58	100.00	98.65	100.00	98.63	100.00	98.30	96.36	97.67	95.77	97.97	95.06	97.20	94.31
$\lambda=\pi$	0	5.18	4.93	9.26	9.42	8.28	8.47	1.47	2.40	6.13	5.18	1.47	2.40	6.13	5.18
	0.010	9.67	9.39	14.91	15.18	14.65	15.01	3.39	4.60	10.80	9.30	3.39	4.60	10.80	9.30
	0.025	29.39	29.62	38.97	40.02	38.75	39.76	16.26	19.20	30.42	27.26	16.26	19.20	30.42	27.26
	0.050	56.78	59.37	72.99	75.27	72.28	74.61	42.75	44.25	56.52	51.52	42.75	44.25	56.52	51.52
	0.100	80.15	85.93	91.14	95.19	90.91	95.08	68.89	65.94	77.91	70.39	68.89	65.94	77.91	70.39
	0.500	94.41	99.99	96.83	100.00	96.72	100.00	91.44	82.98	93.01	83.77	91.44	82.98	93.01	83.77
Joint	0	5.19	4.60	9.27	9.51	8.57	8.73	0.80	1.42	5.31	4.59	0.55	0.68	4.29	2.79
	0.010	9.71	9.49	15.49	15.88	15.22	15.66	1.98	3.44	10.03	8.73	1.40	1.77	8.26	5.85
	0.025	38.25	38.58	44.78	45.73	44.38	45.31	15.72	20.19	37.40	33.73	12.50	13.57	33.64	26.27
	0.050	78.81	80.40	84.65	85.30	84.18	84.81	57.70	61.84	76.92	73.18	53.30	53.24	74.14	66.60
	0.100	97.29	98.27	99.26	99.29	99.20	99.22	91.16	90.61	95.99	93.90	89.07	87.13	95.01	91.56
	0.500	100.00	100.00	100.00	100.00	100.00	100.00	99.93	99.47	99.92	99.49	99.87	98.94	99.93	99.00

Table 2. Rejection frequencies for a random walk drift plus noise model with $\sigma_\eta/\sigma_\varepsilon$=0.5.

		Parametric tests						Nonparametric tests							
		Model known		Model estimated		BSM estimated		Spectral: levels		Spectral: FD		CH: levels		CH: FD	
	$\sigma_\kappa/\sigma_\varepsilon$	Innov.	Smooth.	Innov.	Smooth.	Innov.	Smooth.	m=4	m=8	m=4	m=8	m=4	m=8	m=4	m=8
$\lambda=\pi/2$	0	5.33	4.83	5.33	5.69	4.99	5.35	0.16	0.54	4.90	4.56	0.10	0.27	4.17	3.44
	0.010	7.08	6.60	7.46	7.94	7.07	7.61	0.30	1.02	6.61	6.50	0.24	0.61	5.64	4.64
	0.025	19.11	19.46	22.73	24.38	22.32	24.22	1.98	4.35	18.50	17.66	1.56	2.76	16.80	14.65
	0.050	49.25	52.78	61.01	65.18	61.67	65.66	15.46	22.57	48.37	46.47	13.51	18.16	46.35	41.83
	0.100	79.83	86.35	91.44	95.48	91.64	95.43	52.77	60.19	79.78	77.15	50.13	54.99	78.28	73.97
	0.500	95.77	100.00	98.26	100.00	98.24	100.00	96.92	95.14	97.52	95.48	96.32	93.81	96.94	94.14
$\lambda=\pi$	0	5.04	4.82	9.79	9.98	9.45	9.69	0.35	0.87	5.83	4.91	0.35	0.87	5.83	4.91
	0.010	9.20	8.90	14.65	14.97	14.32	14.55	0.91	1.77	9.66	8.60	0.91	1.77	9.66	8.60
	0.025	26.84	26.54	35.90	36.82	35.48	36.23	5.92	9.60	27.23	24.57	5.92	9.60	27.23	24.57
	0.050	53.92	55.70	68.22	70.61	68.29	70.52	24.40	30.80	53.04	48.54	24.40	30.80	53.04	48.54
	0.100	78.30	83.23	89.99	93.54	90.54	93.51	53.58	56.48	75.69	68.50	53.58	56.48	75.69	68.50
	0.500	95.01	99.98	96.95	100.00	97.04	99.99	89.56	81.93	92.89	83.65	89.56	81.93	92.89	83.65
Joint	0	4.93	4.49	9.68	9.95	9.27	9.48	0.10	0.36	4.99	4.54	0.03	0.12	4.08	2.76
	0.010	8.61	8.22	14.42	14.91	14.27	14.61	0.19	0.69	8.96	7.72	0.09	0.23	7.13	4.76
	0.025	32.04	31.82	38.92	39.91	38.92	39.65	2.93	6.41	31.43	28.24	2.00	3.45	27.84	21.55
	0.050	72.27	73.83	78.72	79.39	78.93	79.71	25.60	36.60	70.26	66.48	21.36	27.01	67.22	59.91
	0.100	95.79	97.10	98.48	98.65	98.16	98.33	72.44	78.29	94.22	91.67	68.82	71.67	92.94	88.96
	0.500	100.00	100.00	100.00	100.00	100.00	100.00	99.74	99.25	99.92	99.47	99.60	98.58	99.89	98.95

Table 3. Rejection frequencies for a random walk drift plus noise model with $\sigma_\eta/\sigma_\varepsilon=1$.

		Parametric tests						Nonparametric tests							
		Model known		Model estimated		BSM estimated		Spectral: FD		Spectral: double FD		CH: FD		CH: double FD	
	$\sigma_\kappa/\sigma_\varepsilon$	Innov.	Smooth.	Innov.	Smooth.	Innov.	Smooth.	m=4	m=8	m=4	m=8	m=4	m=8	m=4	m=8
$\lambda=\pi/2$	0	5.36	4.77	5.58	5.86	6.19	6.43	4.22	4.22	2.35	2.96	3.65	3.29	1.92	2.22
	0.010	8.76	8.11	9.32	9.83	10.06	10.57	7.15	7.05	4.20	5.13	6.17	5.18	3.39	3.71
	0.025	27.18	27.64	31.82	33.62	33.02	34.94	24.11	23.86	18.19	19.98	22.35	19.59	15.79	15.72
	0.050	60.56	64.32	74.27	77.68	75.18	78.57	57.76	55.99	50.73	51.80	55.52	51.45	47.95	46.82
	0.100	86.37	92.50	95.54	98.30	95.59	98.36	85.53	83.07	80.76	79.93	84.15	80.35	79.06	76.93
	0.500	96.78	100.00	98.68	100.00	98.66	100.00	97.70	95.91	95.43	94.34	97.18	94.36	94.61	92.52
$\lambda=\pi$	0	5.13	4.82	8.66	8.81	9.28	9.44	6.16	5.27	7.41	5.87	6.16	5.27	7.41	5.87
	0.010	9.99	9.43	14.83	15.12	15.85	16.16	10.85	9.60	12.53	10.52	10.85	9.60	12.53	10.52
	0.025	30.59	30.82	40.69	41.68	41.78	42.78	30.97	27.80	33.17	29.07	30.97	27.80	33.17	29.07
	0.050	58.03	60.74	74.38	77.03	74.98	77.95	57.15	52.30	58.90	53.12	57.15	52.30	58.90	53.12
	0.100	80.72	86.97	91.31	95.73	91.52	95.91	78.63	71.02	79.90	71.60	78.63	71.02	79.90	71.60
	0.500	94.06	100.00	96.32	100.00	96.38	100.00	93.08	83.82	93.54	84.05	93.08	83.82	93.54	84.05
Joint	0	5.21	4.56	9.49	9.69	10.23	10.53	4.68	4.19	3.67	3.51	3.82	2.64	2.89	2.14
	0.010	10.65	10.06	15.66	16.02	16.79	17.22	9.82	8.85	8.26	7.81	8.18	5.78	6.91	4.85
	0.025	41.15	41.44	47.45	48.25	48.95	49.87	38.70	35.77	35.10	32.90	34.31	27.98	31.12	25.48
	0.050	81.04	82.75	87.35	87.88	87.87	88.42	78.58	75.29	75.91	73.16	75.93	68.87	72.82	66.50
	0.100	97.82	98.77	99.46	99.53	99.46	99.54	96.48	94.47	95.72	93.86	95.62	92.17	94.70	91.15
	0.500	100.00	100.00	100.00	100.00	100.00	100.00	99.93	99.48	99.91	99.34	99.92	99.06	99.81	98.73

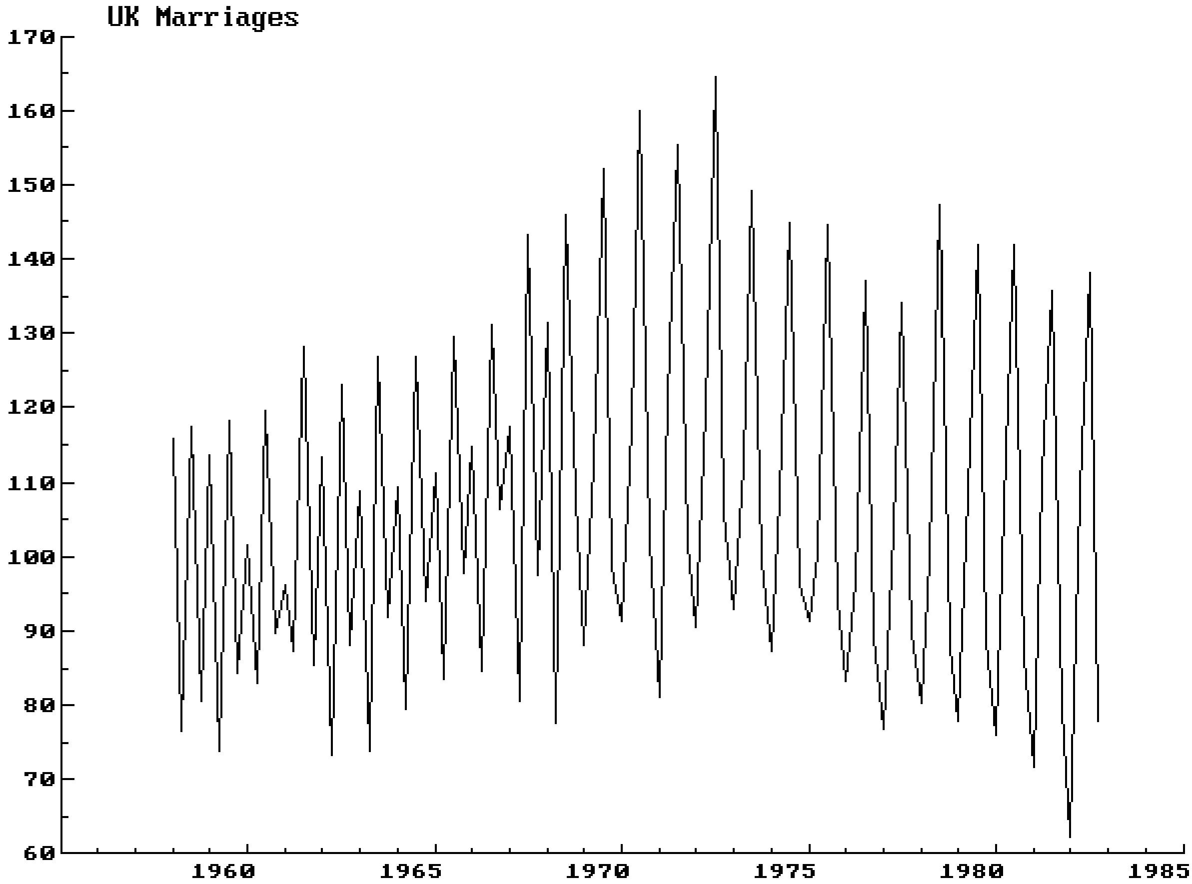
Table 4. Rejection frequencies for a smooth trend plus noise model with $\sigma_\xi/\sigma_\varepsilon$=0.1.

		Parametric tests						Nonparametric tests							
		Model known		Model estimated		BSM estimated		Spectral: FD		Spectral: double FD		CH: FD		CH: double FD	
	$\sigma_\kappa/\sigma_\varepsilon$	Innov.	Smooth.	Innov.	Smooth.	Innov.	Smooth.	m=4	m=8	m=4	m=8	m=4	m=8	m=4	m=8
$\lambda=\pi/2$	0	5.33	4.69	5.53	5.84	5.59	5.90	0.86	1.48	2.58	3.15	0.72	0.96	2.18	2.19
	0.010	8.47	7.63	8.95	9.38	9.05	9.57	1.67	2.65	4.16	5.21	1.40	1.98	3.53	3.79
	0.025	25.64	25.78	30.50	32.16	30.74	32.52	9.62	12.99	17.74	19.08	8.24	9.65	15.46	15.15
	0.050	58.43	61.89	71.84	75.58	71.94	75.88	37.57	42.20	49.42	50.46	34.94	37.30	46.87	45.59
	0.100	85.49	91.48	94.79	97.81	94.70	97.84	73.46	75.22	80.14	79.29	71.77	71.76	78.33	76.27
	0.500	96.55	100.00	98.50	100.00	98.41	100.00	97.30	95.53	95.36	94.26	96.76	93.96	94.62	92.59
$\lambda=\pi$	0	5.16	4.85	8.78	8.96	9.63	9.87	2.77	3.26	7.29	5.77	2.77	3.26	7.29	5.77
	0.010	10.06	9.51	15.07	15.42	16.24	16.51	5.98	6.58	12.36	10.29	5.98	6.58	12.36	10.29
	0.025	30.02	30.11	40.20	41.30	41.63	42.65	22.20	22.92	32.78	28.82	22.20	22.92	32.78	28.82
	0.050	57.88	60.12	74.13	76.49	74.78	77.29	48.58	47.34	58.51	52.94	48.58	47.34	58.51	52.94
	0.100	80.89	86.57	91.24	95.43	91.61	95.66	72.90	68.13	79.73	71.47	72.90	68.13	79.73	71.47
	0.500	94.40	99.99	96.66	100.00	96.88	100.00	92.69	83.56	93.52	83.99	92.69	83.56	93.52	83.99
Joint	0	5.20	4.22	9.47	9.80	10.10	10.37	1.07	1.42	3.67	3.50	0.69	0.73	3.04	2.09
	0.010	10.32	9.40	15.88	16.28	16.67	17.03	2.54	3.63	8.24	7.74	2.03	2.09	6.73	4.78
	0.025	39.61	39.80	46.24	47.09	47.20	48.03	18.81	21.88	34.10	32.14	15.90	15.37	30.36	24.80
	0.050	79.96	81.51	86.02	86.59	86.47	87.06	59.98	62.82	74.86	72.25	55.93	54.82	72.02	65.50
	0.100	97.52	98.50	99.36	99.38	99.37	99.40	91.15	90.81	95.53	93.34	89.11	87.18	94.44	90.94
	0.500	100.00	100.00	100.00	100.00	100.00	100.00	99.88	99.43	99.91	99.34	99.82	98.92	99.82	98.71

Table 5. Rejection frequencies for a smooth trend plus noise model with $\sigma_\xi/\sigma_\varepsilon$=0.5.

		c_D=0	c_D=0.5	c_D=1.0	c_D=1.5	c_D=2.0	c_D=2.5	c_D=3.0	c_D=3.5
	F	4.90	7.98	18.86	39.72	65.21	85.52	95.94	99.26
c_S=0	$\underline{\omega}$	4.90	7.46	16.01	32.90	55.85	77.37	91.33	97.74
	ω	4.89	4.89	4.89	4.89	4.89	4.89	4.89	4.89
	F	9.34	12.94	24.52	44.17	66.36	84.43	94.68	98.75
c_S=1	$\underline{\omega}$	10.53	13.60	23.03	39.08	58.60	77.02	89.81	96.60
	ω	6.08	6.08	6.08	6.08	6.08	6.08	6.08	6.08
	F	24.52	28.19	38.61	53.51	69.58	83.02	92.33	97.10
c_S=2	$\underline{\omega}$	29.07	31.83	39.84	51.65	65.16	77.85	87.75	94.21
	ω	10.32	10.32	10.32	10.32	10.32	10.32	10.32	10.32
	F	45.34	47.74	54.35	63.94	74.19	83.43	90.61	95.34
c_S=3	$\underline{\omega}$	51.46	53.38	58.28	64.92	72.91	80.79	87.45	92.66
	ω	18.13	18.13	18.13	18.13	18.13	18.13	18.13	18.13
	F	62.72	63.97	67.81	73.12	79.32	85.26	90.33	94.16
c_S=4	$\underline{\omega}$	68.83	69.70	72.13	75.80	80.10	84.69	88.87	92.51
	ω	29.22	29.22	29.22	29.22	29.22	29.22	29.22	29.22
	F	74.79	75.58	77.52	80.63	84.07	87.66	91.06	93.96
c_S=5	$\underline{\omega}$	80.10	80.58	81.76	83.56	85.84	88.55	90.97	93.16
	ω	41.62	41.62	41.62	41.62	41.62	41.62	41.62	41.62
	F	89.59	89.82	90.27	90.93	91.87	93.01	94.15	95.31
c_S=7.5	$\underline{\omega}$	92.79	92.84	93.00	93.46	94.02	94.68	95.29	95.97
	ω	68.11	68.11	68.11	68.11	68.11	68.11	68.11	68.11
	F	95.09	95.21	95.25	95.49	95.70	96.05	96.45	96.90
c_S=10	$\underline{\omega}$	97.03	97.05	97.08	97.17	97.27	97.44	97.64	97.87
	ω	83.73	83.73	83.73	83.73	83.73	83.73	83.73	83.73

Table 6. Local asymptotic power against deterministic and/or pure stochastic seasonality of the F, $\underline{\omega}$ and ω tests.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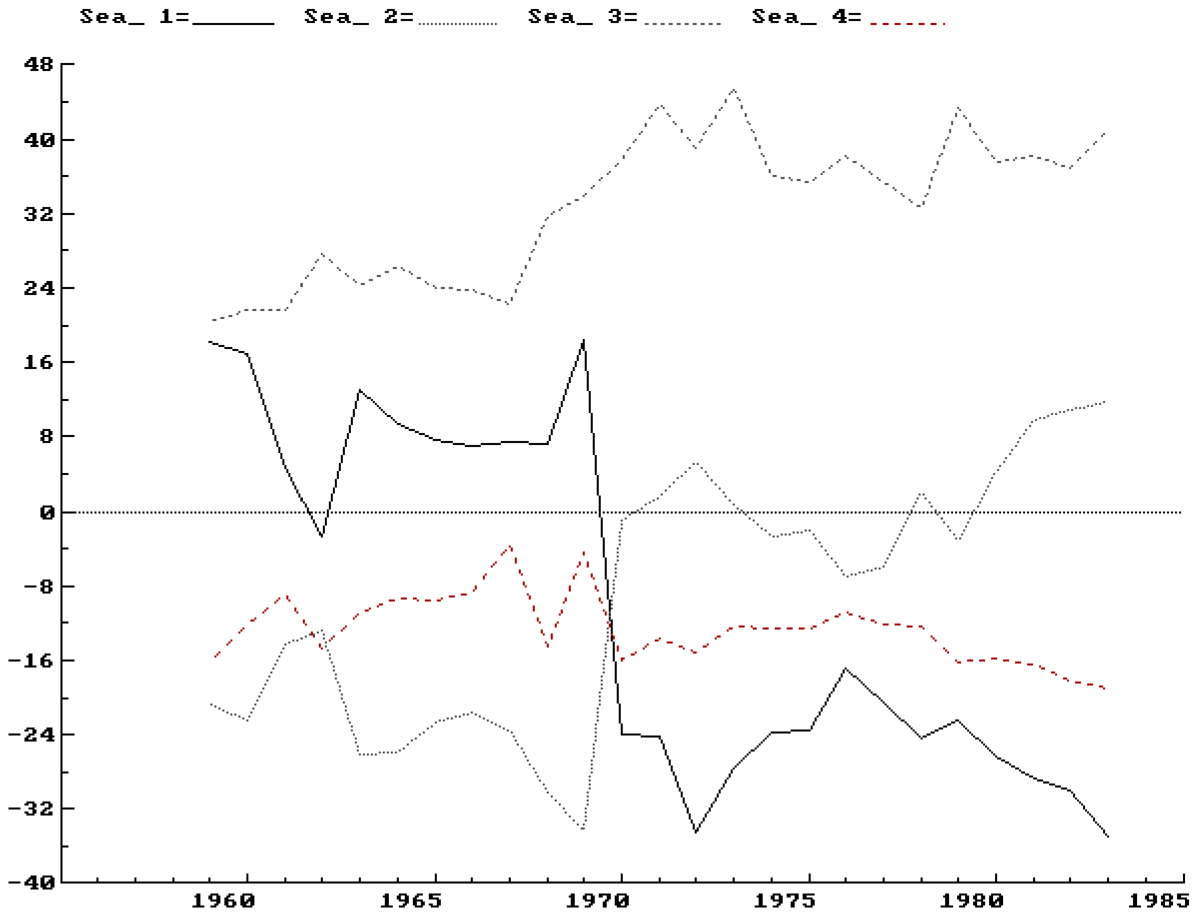

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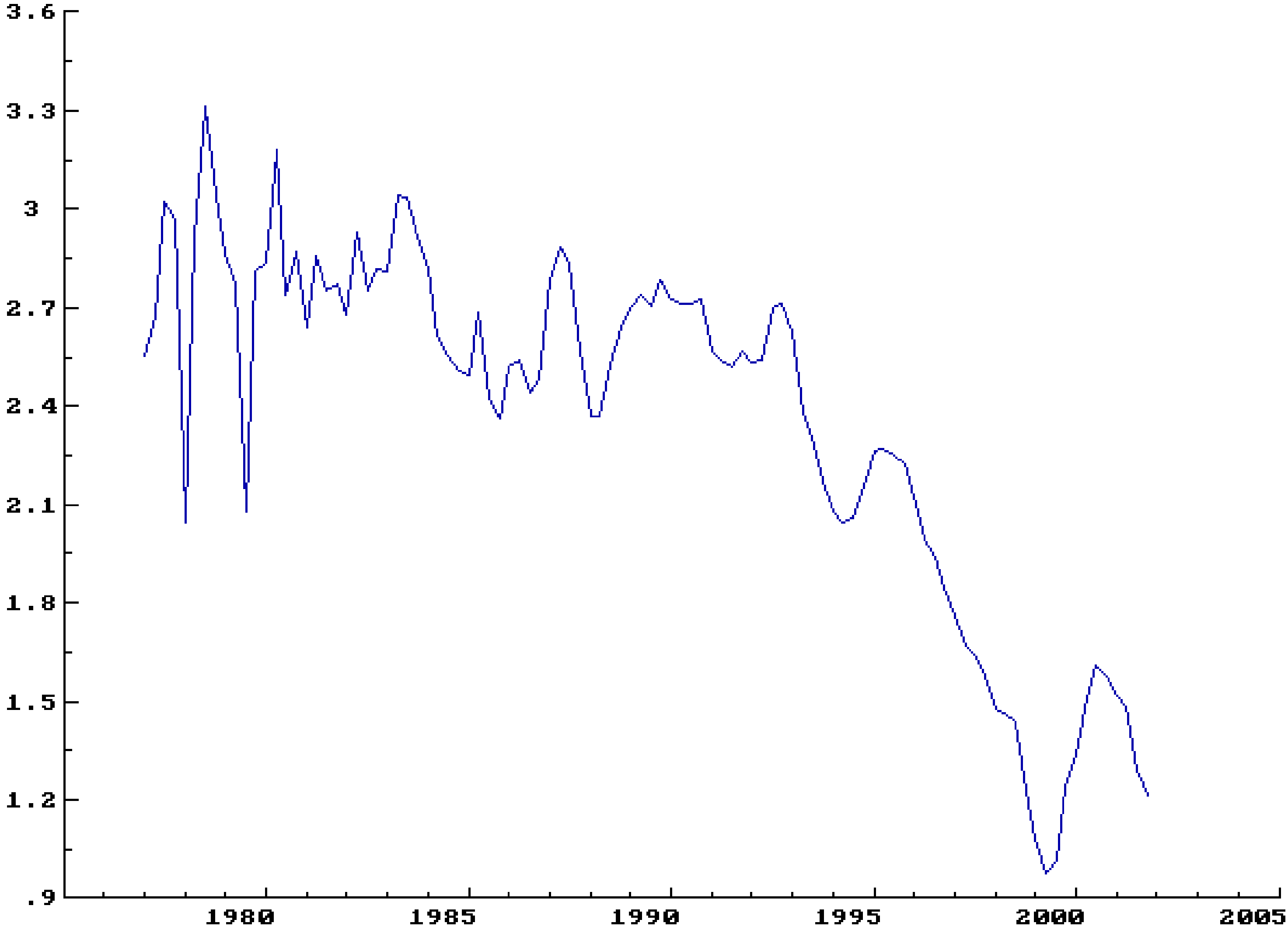
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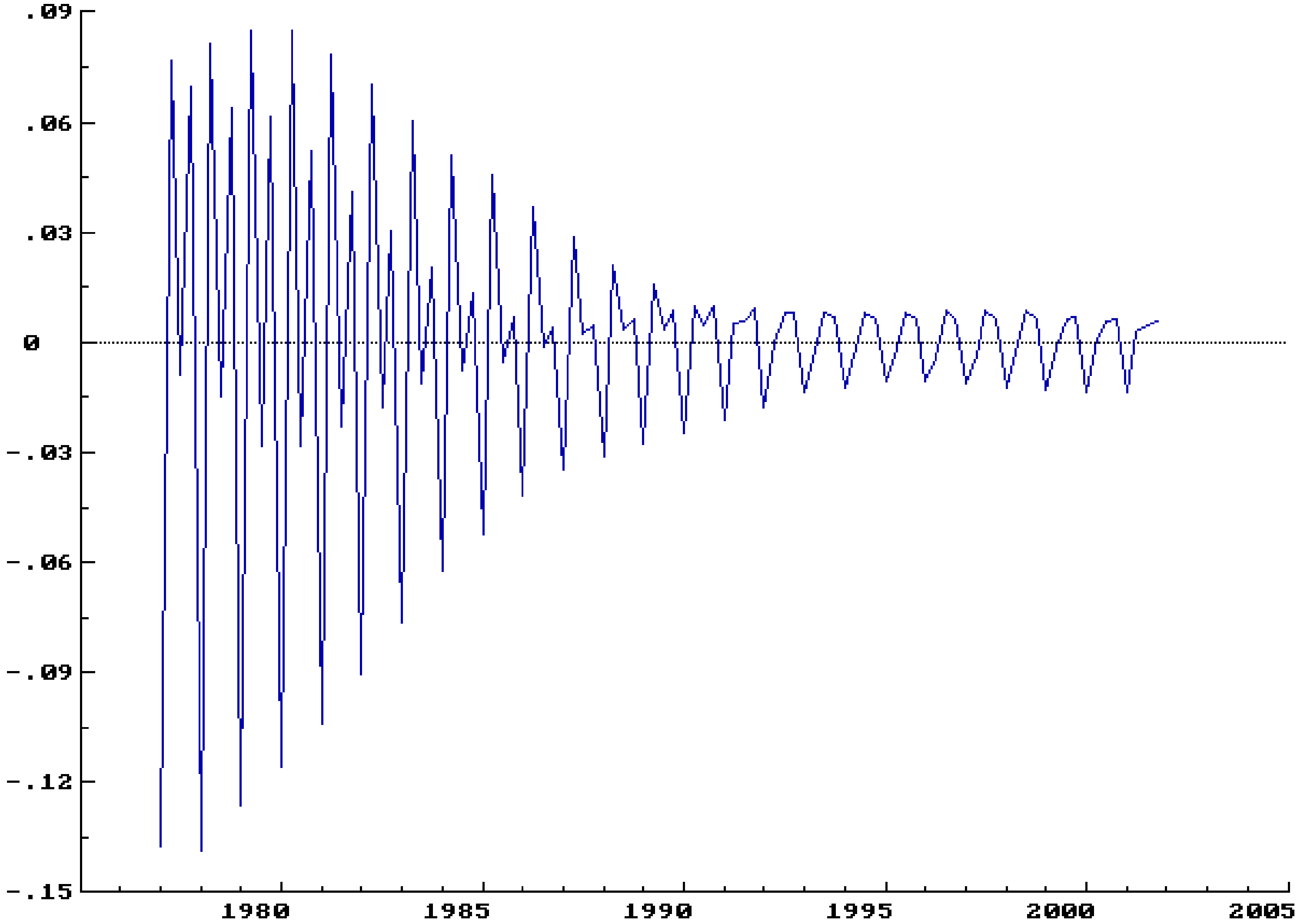

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